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Stock-and-flow-consistent macroeconomic model for South Africa

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Stock-and-flow-consistent macroeconomic model for South Africa

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Abstract:
We develop a stock-and-flow-consistent model for South Africa with four financial instruments and detailed balance sheets for the household, government, financial, non-financial, and foreign sectors and the Reserve Bank. Though micro-founded, the model departs significantly from current dynamic stochastic general equilibrium models as it assumes bounded rationality and no Ricardian equivalence. The stock and flow consistency makes it better suited to studying balance sheet dynamics and the real sector/financial sector interaction. In the model, cyclical flow changes affect the long-term real and financial behaviour of institutions through their impact on the respective institutional assets and liabilities stocks.

Keywords: stock and flow, macroeconomic, model, financial, computable general equilibrium
JEL classification: C68, D53, D58, E44, E47

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1 Introduction

This paper presents a financial-real stock-and-flow-consistent model of the South African economy. The model dynamics build on the simple computable general equilibrium (CGE) model developed by Devarajan and Go (1998) and incorporate elements of dynamic stochastic general equilibrium (DSGE) models and stock-and-flow models in the tradition of Backus et al. (1980) and Godley and Lavoie (2012). The model also incorporates elements of the theoretical models developed by Borio and Zhu (2012) and Woodford (2010).

In recent decades DSGE models have been widely adopted by central banks, finance ministries, and policy analysts; however, they have been subject to extensive criticism, particularly with respect to financial sector dynamics (see Sims 2006; Caballero 2010; Blanchard 2016).[1] In response to these criticisms, there have been significant efforts to incorporate financial dynamics in DSGE models, including the incorporation of financial accelerator mechanisms derived from Bernanke et al. (1999). In these models, a fall in firms' net worth is accompanied by greater reliance on external financing. The mechanism creates a feedback loop between higher lending premiums, associated with the higher agency costs involved in external finance, and falling net worth. The approach is employed by Fernández-Villaverde (2010), Carrillo and Poilly (2010), and Kollmann et al. (2013) to study the impact of fiscal policy. A second modification introducing finance into DSGE models assumes that lenders can force borrowers to repay their loans only in the presence of some durable asset serving as collateral (Kiyotaki and Moore 1997); Ottonello (2013) and Fornaro (2015) study the impact of sudden stops in capital flows with such a model. In the models of Gertler and Karadi (2011) and Ellison and Tischbirek (2014) the ability of a representative bank to borrow from other financial institutions is limited by its balance sheet. The mechanism aims to capture how unconventional monetary policy interventions can reduce balance sheet constraints and increase lending. A different bank lending constraint is used by Gerali et al. (2010), in which the ability of banks to extend loans is limited by the holding of deposits and a capital requirements ratio imposed by the macro prudential authorities.

The inclusion of such financial sector elements in DSGE models creates several problems. The models are linear and thus cannot capture the boom-and-bust dynamics that characterize the financial sector and do not capture heterogeneous and systemic risk, which are important drivers of financial sector dynamics. The inclusion of a financial accelerator mechanism increases the persistence of shocks rather than creating boom-and-bust dynamics (Borio and Zhu 2012; Duca and Muellbauer 2014). Balance sheet dynamics are either not represented at all or considered only for the balance sheet of a representative bank (Gerali et al. (2010) Gertler and Karadi 2011). But, as Calvo et al. (2004), Eggertsson and Krugman (2012), and Borio and Zhu (2012) argue, disaggregated balance sheet dynamics are important for studying

[1] Two milestones in the development of DSGE models have contributed to their adoption for policy analysis. The seminal work of Smets and Wouters (2003) was the first to estimate a micro-founded DSGE model using Bayesian estimation and use it to forecast. The model consists of seven variables (GDP, consumption, investment, prices, real wages, employment, and the nominal interest rate) and ten structural shocks (including productivity, labour supply, investment preferences, cost-push, and monetary policy shocks). Christiano et al. (2005) introduced several variations designed to account for aspects of economies that policymakers face: habit formation in consumer preferences; adjustment costs in investment; variable capital utilization; and firms' requirement for credit as working capital to finance their wage bill. They also showed that an optimisation-based model with nominal and real rigidities can account successfully for the effects of a monetary policy shock.

the impacts of sudden stops, fiscal policy, and general risk behaviour of agents in the economy.

To address some of these criticisms, we develop a model that is stock-and-flow-consistent.[2] This implies that we have several financial instruments, rates of return, and institutional balance sheets. We model equities, bonds, loans, and cash and deposits as financial instruments; their returns; and the balance sheets of the Central Bank, the household sector, the financial sector, government, the non-financial sector, and the foreign sector. This is a significantly richer representation than the financial representation of institutions and financial instruments in DSGE models. The stock and flow consistency implies that there are strict budget constraints. Changes to the balance sheet of one institution must be matched by changes to the balance sheets of other institutions. These changes reflect that some institutions save more than they invest in physical capital and thus increase their net financial assets. At the same time, those institutions that record higher investment in physical capital than their savings see an increase in their net financial liabilities. The changes to the balance sheets also reflect changes to the prices of assets and liabilities. What is a particularly striking difference between stock-and-flow-consistent models and other models is that cyclical flow changes affect the long-term real and financial behaviour of institutions through their impact on the respective institutional assets and liabilities stocks (Backus et al. 1980).

Recent analyses using stock-and-flow-consistent models include Barwell and Burrows' (2014) study of the evolution of the UK economy in the years leading up to the financial crisis of 2008, in which balance sheet linkages enable financial fragilities to be identified. In their stock-and-flow-consistent model, Caiani et al. (2014) analyse the monetary dynamics that emerge from a Schumpeterian structural change in the economy driven by innovation. Burgess et al. (2016) develop a stock-and-flow-consistent model for the United Kingdom and use it to study the impact of house price changes, shocks to the risk-weighted capital ratio, government consumption shocks, and sudden-stop shocks. They also highlight some of the problems associated with stock-and-flow-consistent models as compared with DSGE models. These problems include model equations which are not based on the optimization problem of individual agents (making the model parameters subject to the Lucas Critique), high levels of complexity due to the requirement for stock and flow consistency, and large data requirements.

While our model is similar in terms of its stock and flow consistency to those recent models, it is different in terms of the behavioural specification for the different agents. Consumption and production behaviour are micro-founded in agents' inter-temporal optimization, allowing us to capture how changes in preferences, technology, and resource constraints affect outcomes. Prices exhibit a degree of stickiness, and there is a monetary policy reaction function based on a Taylor Rule. These features make it similar to new Keynesian DSGE models, but unlike DSGE models ours is not stochastic.

There are two features of our model that make it different to both the traditional-stock-and-flow-consistent models and DSGE models. These features provide better representation of financial sector dynamics. First, our analysis of financial sector behaviour is based on modern theories of financial transmission mechanisms developed in the wake of the 2008 global financial crash (Woodford 2010; Borio and Zhu 2012), with modifications appropriate for

[2] Caverzasi and Godin (2015) provide a comprehensive review of the evolution of stock and flow models and their application to economic questions. Godley and Lavoie (2012) compare the characteristics of the stock and flow models developed in the tradition of Backus et al. (1980) and those that are similar to Godley (1996).

application to South Africa. Second, we specify a dynamic adjustment model of household expectations with properties that differ radically from the way expectations are formed in both stock-and-flow-consistent and DSGE models.

1.1 Financial sector behaviour

In the model developed by Woodford (2010), financial intermediaries' lending spread is a function of financial sector capital. Raising the level of capital is costly and leverage is limited by regulatory requirements. Shocks that impair the capital of the intermediary or the regulatory requirements for a higher leverage ratio translate into higher lending spreads, lower volumes of lending, and reduced economic activity. Borio and Zhu (2012) also link the capital of the financial sector to bank behaviour. In their framework, behaviour is driven by the *capital threshold effect* and the *capital framework effect*. The capital threshold effect arises because breaching the minimum threshold is costly for a bank. In the face of a possible breach, banks will take defensive action to avoid these high costs, which will affect the availability and pricing of funding extended to customers. The capital framework effect influences the way the banks measure, manage, and price risk, which affects their behaviour. The economic cycle changes the strength of the capital threshold effect as probabilities of default, valuations, and the perception of risk change. In turn, this shifts the relative position of the banks' capital to the regulatory threshold and affects bank behaviour. The financial accelerator effects in both models are driven by the relationship between capital and economic activity. Higher economic activity reduces the probability of default and the perception of risk, hence improving valuations. This reduces lending spreads, which encourages further improvements in economic activity.

1.2 Household expectations

The representative household in our model has model-consistent (rational) expectations over a short period of time (ten periods). The consumer does not have perfect foresight, unlike in DSGE models, which assume rational expectations. Borio and Zhu (2012) argue that rational expectations effectively imply perfect foresight of risk, which hinders the incorporation of cross-sectional and inter-temporal co-ordination failures. This implies that rational expectations models, at least as currently operationalized, are less suited to studying financial sector dynamics. In existing stock-and-flow-consistent models, expectations are generally adaptive, meaning that the consumer has no foresight even over a finite period of time (Godley and Lavoie 2012). Both adaptive and rational expectations models have been criticized extensively.[3]

In our model, the representative household finds a solution for the current period by inter-temporally optimizing over the next ten periods. Once a solution is found, the next period becomes the current period and the household needs to find a solution again based on model-consistent expectations. This allows for the expectations process to change between periods and different rules to be introduced. For example, instead of optimizing over ten periods, the household can optimize over five periods. This renders the model suitable for analysing non-linearities such as sudden-stop shocks, and it is in line with recent research, which indicates that households tend to have bounded rationality (Hommes 2011; Roos and Luhan 2013). Bounded rationality is based on the seminal work of Simon (1955),[4] who argues

[3] See for example Gertchev (2007).

[4] See also Simon (1982) and Simon (1986).

that economic agents do not follow a common, invariant basic process in making decisions; rather, their expectations depend on what the situational context is, how it emerges, and how reasoning operates within this context.

In the next section, we present the model. This is followed by a section on how the balance sheets for the various institutions are generated and the data used in the model. In Section 4, we present the calibration approach. The baseline produced by the model is presented in Section 5, and Section 6 concludes.

2 Detailed model description

Our model comprises a small general equilibrium model with financial stock and flow dynamics.[5] There are six types of institution that make real and financial decisions:

- the representative household
- the representative firm (non-financial corporation)
- the representative financial corporation
- government
- the Central Bank, and
- the rest of the world.

The financial instruments are grouped in five categories: equities, loans, cash and deposit, bonds, and other. Figure 1 presents the conceptual framework.

The different agents meet in the financial, product, and factor markets.

In the financial market, decisions are made regarding the accumulation of financial assets and liabilities. These are represented by the maroon lines labelled 'Changes in the stock of assets and liabilities' in Figure 1. There are sub-markets for money, bonds, equities, and loans. The markets are linked through a set of asset demand functions and stock and flow equilibrium rules, which ensure that stock and flow consistency is always maintained. The markets for bonds and loans are cleared through their rates of return, while the markets for money and equities are cleared *on demand*. The financial sector and the non-financial sector provide money and equities to ensure that demand is equal to supply. The rates of return are affected by the policy rate, which is determined by the Reserve Bank. It is assumed that the monetary authorities follow a Taylor Rule (the black line labelled 'Taylor Rule').

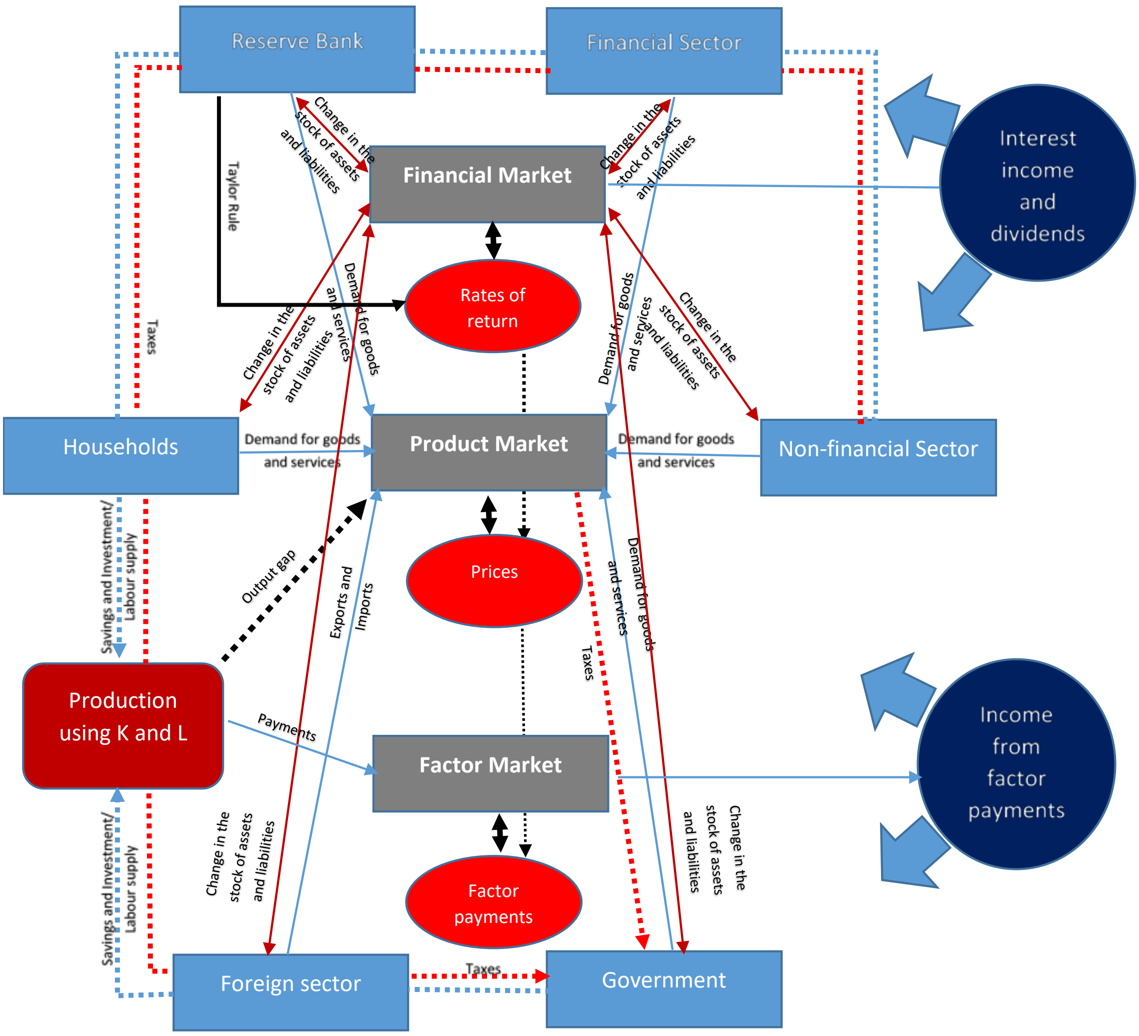
The financial market also distributes net dividend and interest income (represented by the large dark-blue circle) to all institutions.

Changes in an agent's asset portfolio are equal to changes in their financial wealth. This, in turn, is a function of financial wealth from the previous period, capital gains, changes to the stock of liabilities, and net saving. Decisions to invest in financial assets are driven by the level of economic activity as well as the rates of return and costs associated with holding liabilities. The foreign and financial sectors make their decisions based on relative returns, following Tobin asset demand functions (Backus et al. 1980).

[5] All the model equations, variables, and parameters are presented in Appendix A.

In the product market, the supply of goods and services is driven by producers maximizing profits subject to a constant elasticity of substitution (CES) production function. We have one domestically produced good. Demands arise from the household, government, investment, and net exports. These are represented by the blue lines labelled 'Demand for goods and services'. Prices of imports, exports, and the domestically produced good adjust to ensure flow equilibrium in the product market. These are represented in the diagram by the red circle labelled 'Prices'.

Figure 1: Diagrammatic representation of the model framework



Source: Authors' elaboration.

In the factor markets, the demands for capital and labour are driven by the real borrowing costs in the economy as well as the deviation of aggregate demand from its steady state. The economy-wide production function represented by the maroon square employs the factors of production and makes factor payments, which are distributed to the capital owners and labour (the second dark-blue circle, labelled 'Income from factor payments'). The real borrowing costs reflect the prevailing credit conditions and, along with aggregate demand, proxy the current economic conditions. Higher real rates reduce the demand for factors of production directly and indirectly through their impact on aggregate demand. Labour demand tends to be more sensitive to changes in real borrowing costs and aggregate demand than capital, as capital is generally activity-specific and thus immobile. The factor returns adjust reflecting the imperfect substitutability of capital and labour. These are represented by the red circle labelled 'Factor payments'. Under-utilization of production factors represents a negative output gap and lower supply of goods and services than the potential of the economy (the black dotted line labelled 'Output gap'). The level of investment by each institution (the dotted blue line) determines the stock of capital employed in the production function. Labour is assumed to grow at an exogenous growth rate.

The three markets are linked through:

1. The impact of balance sheet changes on real lending rates and the subsequent effect on the demand for goods and services and factors of production. This channel works primarily through the net worth of the financial sector, the lending spread, investment, and household consumption. This also affects the demand for factors of production.
2. Inflation and its impact on asset prices and monetary policy decisions.
3. Financial assets, which generate dividend and interest income. These and other income sources generate demand for goods and services as well as demand for financial assets and liabilities in the next period.
4. And finally, real economic activity and its impact on asset prices and demand for assets and liabilities and on factors of production.

Similarly to Devarajan and Go (1998), the model includes three macroeconomic balances: the government balance, the external balance, and the savings–investment balance. These are in addition to accounting rules that ensure stock and flow consistency on the financial side.

The financial sector provides intermediation services. Its demand for assets is represented by a Tobin asset demand function. Its decisions to accumulate assets and liabilities and hold reserve assets drive the lending spread and contribute to equity price growth. The financial sector's stock of reserve assets and its desired reserve asset ratio is a main determinant of the credit multiplier, a key component of our financial accelerator mechanism.

The firm's (non-financial corporation) sector is responsible for the bulk of investment in the economy, which is driven by a Tobin's Q specification. To finance its investment expenditure and demand for financial assets it issues equities on a perfectly elastic supply function; thus the amount of equity finance is driven by savers' demand for equities.

Government receives direct and indirect taxes (represented by the dotted red lines) in addition to factor income, dividends, interest income, social contributions, and other income. Government consumption expenditure is determined by a discretionary growth rate. The change in the stock of bonds issued by the government closes the government flow balances.

The external sector interacts with the domestic economy in both the financial and the product markets. Exports and imports are modelled as imperfect substitutes for the domestically produced good and are driven by changes in relative prices. Some of the foreign liabilities of the domestic economy are fixed in foreign currency units, while others vary with the level of domestic economic activity. The exchange rate ensures the closure of the external balance. It also affects the liability side of the foreign sector (expressed in local currency units) and, along with exogenous changes to foreign savings, leads to changes in the financial wealth of the external sector.

The representative household maximizes consumption subject to a future wealth target and all equations in the model. The target is modelled as exogenous, given its current level of real wealth. An exogenous growth rate indicates the real wealth that the representative household chooses to achieve in the future. For simplicity, the wealth target assumption is adopted in the tradition of Pigou's real wealth effect (Patinkin 1948; Tobin 1975).[6] The household receives factor income, dividends, interest income, social contributions, and other income. It makes decisions about consumption (savings), investment, and asset and liability accumulation.

The model assumes that savings by the financial and non-financial sectors adjust to ensure that the savings–investment balance is maintained.

What follows is a detailed representation of the model.

2.1 Producer behaviour

The modelling of production, exports, and imports closely follows Devarajan and Go (1998). The representative firm maximizes a CES production function subject to a given set of input and output prices. We assume constant returns to scale:

$$QVA_t = \alpha^p \cdot \left(\sum\nolimits_f \delta_f^p \cdot QF_{tf}^{-\rho^p}\right)^{-1/\rho^p} \tag{1}$$

where *QVA* is the value added in period t, α^p is a shift parameter reflecting total factor productivity (TFP), *QF* is the quantity demanded of each factor *f* (i.e., labour and capital), and δ^p is a share parameter of factor *f* employed in the production process. The elasticity of substitution between factors σ_i^p is a transformation of ρ^p (i.e., $\sigma_i^p = 1/(1+\rho_i^p)$). The factor demand equation is:

$$QF_{tf} = \alpha^{p^{-\frac{\rho^p}{1+\rho^p}}} \cdot QVA_t \cdot \left(\delta_f^p \cdot \frac{PVA_t}{WF_{tf}}\right)^{1/(1+\rho^p)} \tag{2}$$

Intermediate inputs are a fixed share of valued added. Total aggregate output is equal to the sum of value added and intermediate demand.

[6] In their stock-and-flow-consistent models, Godley and Lavoie (2012) employ a specification of level of wealth expected at the end of the period that is based on the actual wealth in the previous period plus the level of expected savings. The level of expected wealth is a driver of the demand for assets. In our specification, the target level of wealth also drives the household demand for assets. However, the target is a function of an exogenous growth rate and real wealth achieved in the previous period. Households also optimise inter-temporally, unlike in the models developed by Godley and Lavoie. The growth rate is assumed to be exogenous in order to simplify the dynamics of the model. An endogenous growth rate would provide an additional channel for households to respond to the shocks in the macroeconomy.

2.2 Behavioural functions governing international trade

Imports are modelled using an Armington specification (Armington 1969). Imported and domestically produced goods are imperfect substitutes. Changes in the relative price of imports lead to a change in the ratio of imports to domestic sales:

$$QQ_t = \alpha^q \left[\delta^q \cdot QD_t^{-\rho^q} + (1-\delta^q) \cdot QM_t^{-\rho^q}\right]^{-1/\rho^q} \tag{3}$$

$$(1 - ts_t) \cdot PQ_t \cdot QQ_t = PD_t \cdot QD_t + PM_t \cdot QM_t \tag{4}$$

$$PM_t = (1 + tm_t) \cdot pwm_i \cdot EXR_t \tag{5}$$

where ts_t is an indirect sales tax, α^q is a shift parameter, δ^q is a share parameter, *QQ* is the composite good consumed domestically at time t, *QD* and *QM* are domestically supplied and imported quantities, and *PD* is the price of domestic good *QD*. The import price *PM* is determined by world import prices *pwm*, import tariff rates *tm*, and the exchange rate *EXR*; under the small-country assumption, *pwm* and *tm* are exogenous variables. PQ_t is the composite supply price.

A constant elasticity of transformation (CET) function determines the relationship between the quantity of goods produced for domestic and foreign export markets:

$$QA_t = \alpha^t \left[\delta^t \cdot QD_t^{\rho^t} + (1-\delta^t) \cdot QE_t^{\rho^t}\right]^{1/\rho^t} \tag{6}$$

$$PA_t \cdot QA_t = PD_t \cdot QD_t + PE_t \cdot QE_t \tag{7}$$

$$PE_t = (1 - te_t) \cdot pwe_t \cdot EXR_t \tag{8}$$

where *QE* is the quantity of exports, *te* is the export tax rate (negative if a subsidy), *pwe* is the exogenous world export price, α^t is a shift parameter, and δ^t is a share parameter. PE_t is the export deflator, which is a function of the world prices, the exchange rate, and the export tax.

The above equations lead to the following first-order conditions which define the ratio of *QD* to *QM* and the ratio of *QD* to *QE*:

$$\frac{QD_t}{QM_t} = \left(\frac{\delta^q}{1-\delta^q} \cdot \frac{PM_t}{PD_t}\right)^{1/(1+\rho^q)} \tag{9}$$

$$\frac{QD_t}{QE_t} = \left(\frac{\delta^t}{1-\delta^t} \cdot \frac{PE_t}{PD_t}\right)^{1/(\rho^t-1)} \tag{10}$$

2.3 Prices

In this section, we outline the specifications driving prices other than *PE* and *PM*, which were described above.

$$PQ_t = (1 + inf) \cdot PQ_{t-1} + \theta_1^{pq} \cdot \left(y_{t-1}^{gap,l} - 1\right) + \theta_2^{pq} \cdot \Delta PM_t \qquad (11)$$

PQ is the supplier price excluding sales taxes. It is also the numeraire price in our framework. inf is the target rate of inflation, which also proxies steady-state inflation. The proxy variable for the output gap $y_{t-1}^{gap,l}$, which is described below, and the change in import prices ΔPM_t affect inflation. The coefficient θ_1^{pq} measures the responsiveness of prices to the output gap, while the coefficient θ_2^{pq} measures the responsiveness of prices to changes in import prices. The equation reflects Phillips curve dynamics.

The sales price including sales taxes is related to *PQ* via the simple identity:

$$PRC_t = (1 + ts_t) \cdot PQ_t \qquad (12)$$

The price of domestically produced output *PD* is determined in Equation 13. It is a function of the nominal value of the composite output, nominal imports, and real output of the domestically produced good.

$$PQ_t \cdot QQ_t = PD_t \cdot QD_t + PM_t \cdot QM_t \qquad (13)$$

The activity price *PA*, which is inclusive of activity taxes *ta*, is defined similarly in Equation 14.

$$PA_t \cdot QA_t = PD_t \cdot QD_t + PE_t \cdot QE_t \qquad (14)$$

The value-added price is a function of the nominal activity output after tax minus the nominal value of intermediates over real value added.

$$PVA_t \cdot QVA_t = PA_t \cdot (1 - ta_t) \cdot QA_t - PRC_t \cdot QINTA_t \qquad (15)$$

We model inflation expectations as adaptive. Chow (2011) provides strong econometric support for adaptive expectations.[7] Equation 16 specifies how price expectations are formed. The coefficient θ^p captures the response of expectations to deviations of expected prices from actual prices. More credible monetary policy implies a smaller coefficient. The coefficient θ^{prce} measures the sensitivity of expectations to changes in the output gap and is significantly smaller than the coefficient θ_1^{pq}. While inflation expectations are directly affected by the level of economic activity, this impact is significantly smaller than the direct impact on inflation.

[7] Also, Kabundi and Schaling (2013) find that the formation of inflation expectations tends to be adaptive in South Africa.

$$PRC_t{}^e = (1 + inf) \cdot PRC_{t-1}{}^e - \theta^p \cdot (PRC_{t-1}{}^e - PRC_{t-1}) + \theta^{prce} \cdot \left(y_{t-1}^{gap,l} - 1\right) \quad (16)$$

Empirical evidence does not support assumptions of rational expectations (Amano et al. 2011; Dias et al. 2010; Johannsen 2014; Mankiw et al. 2003). For South Africa, Kabundi and Schaling (2013) also find that inflation expectations are not consistent with the rational expectations.

The equity price is a function of the steady-state growth rate ss^{peq}, which is calibrated as the trend growth rate over the calibration period; expected inflation, which affects the equity price with elasticity μ_1^{peq}; the change in the stock of money created by the financial sector $SL_t^{fin\_cd}$ (a proxy for money supply growth), which affects the equity price with elasticity μ_2^{peq}; and the change in aggregate output, with elasticity μ_3^{peq}. In empirical studies these have been identified as important drivers of equity prices (Chen et al. 1986; Rapach et al. 2005. For South Africa, Gupta and Modise (2013) find that interest rates, money supply, and growth of world oil production affect stock prices.

$$PEQ_t = \left((1 + ss^{peq}) \cdot (1 + \pi_t^e)^{\mu_1^{peq}} \cdot (\Delta SL_t^{fin\_cd})^{\mu_2^{peq}} \cdot (\Delta QA_t)^{\mu_3^{peq}}\right) \cdot PEQ_{t-1} \quad (17)$$

2.4 Investment and savings

Most of the investment in our framework is done by the firm (non-financial corporation). We model investment in fixed capital and we keep investment in inventories exogenous. The investment function is

$$I_t = I_{t-1} \cdot \gamma_1^I \cdot \left(\frac{\Delta PEQ_t}{\Delta PRC_t}\right)^{\gamma_2^I} \cdot \left(\frac{1 + r_t^l}{1 + \pi_t^e}\right)^{\gamma_3^I} \quad (18)$$

Investment in every period is linked to past investment through the fixed coefficient γ_1^I, which reflects trend growth. In addition, investment in the current period varies with the change of the ratio of equity prices to sales prices and the real rate on loans; γ_2^I and γ_3^I are elasticities which measure the responsiveness of investment to the two terms. The term $\left(\frac{\Delta PEQ_t}{\Delta PRC_t}\right)$ captures Tobin's Q effects. Higher equity prices relative to prices of goods and services (a proxy for the book value of the firm) lead to higher levels of investment. Investment by other institutions such as the household and the financial sector is modelled similarly without the Tobin's Q effects.

The savings of the household sector are defined as after-tax income minus household consumption and other expenditure:

$$SAV_t^H = (1 - td_t^H) \cdot PRC_t \cdot YI_t^H - SC_t^H - OI_t^H - IEXP_t^H - PRC_t \cdot QH_t \quad (19)$$

where $YI_t^H, SC_t^H, OI_t^H, IEXP_t^H$, and QH_t are respectively real household income, social contributions made by the household, other contributions, interest rate expenditure, and real household expenditure.

Savings for the financial and non-financial sector adjust to ensure that the savings and investment constraint is maintained. This adjustment mechanism is a departure from the stock-and-flow-consistent model developed in the tradition of Godley and Lavoie in which the adjustment takes place through investment in inventories.

$$SAV_t^{ii} = MPS_t^{ii} \cdot (1 - td_t^i) \cdot PRC_t \cdot YI_t^{ii} \tag{20}$$

The set *ii* consists of the financial and non-financial sectors. MPS_t^{ii} is the marginal propensity to save, defined as:

$$MPS_t^{ii} = mpsbar^{ii} \cdot (1 + MPSADJ_t) \tag{21}$$

The steady-state marginal propensity $mpsbar$ varies, driven by the term $MPSADJ_t$. Savings of the foreign sector are exogenous, while savings of the Central Bank are simply the after-tax income left after paying for interest expenditure.

2.5 Financial behaviour

The financial behaviour in our framework is based on the flow-of-funds dynamics. In every period, agents experience a change in their financial wealth. This is driven by their decisions to accumulate liabilities (sources of funds), changes to the equity price, and net savings. The financial wealth is then divided into different financial assets. Institutions that save more than they invest have net incurrence of financial assets which exceeds the net incurrence of financial liabilities. The opposite is the case if investment exceeds savings.

The equation defining financial wealth is:

$$\begin{aligned} FW_t^i = \sum_{fi} SA_{t-1}^{i,fi} + PEQ_t \cdot SA_{t-1}^{i,eq} + \sum_{fi} dl_t^{i,fi} + PEQ_t \cdot dl_t^{i,eq} \\ + SAV_t^i - PRC \cdot I_t^i + n_t^i \end{aligned} \tag{22}$$

where *i* is a set of agents consisting of household, government, Central Bank, financial sector, non-financial sector, and the rest of the world. The set *fi* consists of the financial instruments cash and deposits, loans, bonds, and other assets. We do not model prices for these instruments. We simplify our analysis by assuming that agents hold bonds to maturity. *SA* is the stock of assets; *dl* is a flow variable representing the change in the stock of liabilities. Changes to equity prices increase the value of equities and the funds available for financial investment. $SA_{t-1}^{i,eq}$ is the stock of equities in the previous period.

Next, we discuss how the various institutions choose to divide their financial wealth across the different financial assets and how they increase their liabilities.

2.6 Assets

The asset demand specification for the financial and foreign sectors is based on a Tobin asset demand function (Backus et al. 1980; Godley and Lavoie 2007; Tobin 1982). The general specification is:

$$SA_{fii,t}^{it} = (FW_t^{it} - SA_t^{it,l}) \cdot \left(\lambda_{\text{fii},0} + A_{fii,j} \cdot r_{fii,j} + \lambda_{\text{fii},4} \frac{PRC_t \cdot YI_t^{it}}{FW_t^{it}}\right) \tag{23}$$

where the set *it* consists of the foreign and financial sectors, the set *fii* has the financial instruments equities, bonds, and cash and deposits. $\sum_{fii} \lambda_{\text{fii},0} = 1$, $\sum_{fii} \lambda_{\text{fii},4} = 0$; A is a matrix of coefficients which satisfies $\sum_{fii} A_{\text{fii,j}} = 0$; j is equal to the number of financial instruments, in this case three; and $\sum_j A_{\text{fii,j}} + \lambda_{\text{fii},4} = 0$. The coefficients of matrix *A* show the responsiveness of the holding of a financial instrument as an asset to changes in its own return as well as in the returns of other assets. An increase in the return of equities, which is a function of equity prices, and current-period dividend payments, relative to the return on money and bonds, increases the demand for equities and their relative share in the financial sector portfolio. This is at the cost of reducing the shares of money and bonds. This ensures that the share of one asset can increase only if the shares of other assets fall. The stock of loans $SA_t^{it,l}$, which is determined outside the Tobin function, is subtracted from the total financial wealth *FW*. The coefficient $\lambda_{\text{fii},4}$ reflects the transactional demand for money, which is represented by the cash and deposit instrument in our framework. An increase in the share of nominal income relative to financial wealth should translate into a higher share of cash and deposit holding.

The stock of loans provided by the financial sector to the economy is a function of the deposits held by the financial sector in the previous period. Higher cash and deposits liability in the previous period translates into higher loans in the current period.

$$SA_t^{fin,loans} = (1 - RR_t) \cdot SL_{t-1}^{fin,cashdep} \tag{24}$$

The ratio RR_t is key to the operations in our model. It reflects the requirement by the Central Bank for financial institutions to hold cash reserves, but it also reflects the financial sector's willingness to hold reserves for other reasons, such as to manage liquidity or risk. This aims to capture the mechanism identified by Borio and Zhu (2012). A decrease in the ratio can reflect higher willingness to take risk. A reduction in the ratio increases the supply of loans and reduces the lending spread. This in turn encourages investment in the economy and the building of capital stock. The RR_t ratio provides a link between the behaviour of the financial sector and the real economy. At the same time, developments in the real economy affect the ratio through the repo rate and the growth in the financial assets of the financial sector. The relationship in Equation 24 also represents a money multiplier.

The ratio is calibrated by dividing the stock of loans on the asset side of the financial sector by the stock of cash and deposits on the liability side. Movements in the repo rate reflect changes in the cycle, which affect asset prices and the net worth of agents in the economy, including the financial sector. The repo rate affects bank lending directly and indirectly through the financial net worth. The growth in financial assets captures the prevailing financial conditions and is sensitive to changes in capital requirements. The introduction of higher capital ratio can lead to repricing of risk, reduction in the growth of financial sector assets, and an increase in RR_t. Changes to the ratio affect all institutions through the loan supply by the financial sector and the lending spread. This in turn affects assets prices and economic activity, creating feedback loops that operate through the balance sheets of all agents.

β^{repo} is the responsiveness of the reserve ratio to changes in the repo rate, whereas β^{sa} is the responsiveness to changes to growth in the balance sheet of the financial sector; *fi* is a set including all financial instruments. Equation 25 outlines the specification for the reserve ratio.

$$RR_t = RR_{t-1} \cdot \left(1 + \Delta r_{t-1}^{repo}\right)^{\beta^{repo}} \cdot \left(\frac{\sum_{fi} SA_t^{fin,fi}}{\sum_{fi} SA_{t-1}^{fin,fi}}\right)^{\beta^{sa}} + \lambda_t^{rr} \tag{25}$$

This specification also aims to capture the mechanism identified by Woodford (2010). In his model, the investment/saving (IS) curve links the loan spread to the demand and supply of intermediary services and the level of economic activity.[8] The supply of intermediary services, in turn, depends on the capital of intermediaries as well as on factors that can loosen or tighten the leverage constraint, such as changing attitudes of intermediaries' creditors. Improvements to the net worth of the financial sector, for example, can increase the level of intermediation services, reduce the spread, and increase economic activity. In our framework, the economic activity is captured by the repo rate, the attitudes of intermediaries' creditors is captured by the exogenous parameter λ_t^{rr}, and the term $\left(\frac{\sum_{fi} SA_t^{fin,fi}}{\sum_{fi} SA_{t-1}^{fin,fi}}\right)^{\beta^{sa}}$ accounts for balance sheet effects.

If the relationship between output and interest rates is elastic, the model framework can create financial accelerator effects.

The provision of loans by the rest of the world as well as by non-financial companies and government is a function of the financial wealth in the previous period and the repo rate. A higher repo rate decreases the share. This indicates that with increases in the repo rate, the economy is likely to slow down and the credit-worthiness of borrowers to deteriorate.

The demand for cash and deposits by the household, the non-financial sector, and government is driven by the real rate on cash and deposits as well as by the nominal income of each institution. Higher real rates increase the demand for deposits as a store of value, while higher income increases the transactional demand:

$$SA_t^{il,cd} = \alpha^{il,cd} \cdot \left(\frac{1 + r_t^{cd}}{1 + \pi_t}\right)^{\mu^{cd}} \cdot PRC_t \cdot YI_t^{il} \tag{26}$$

where *il* is the set of agents, α is the steady-state coefficient which links the stock of cash and deposit assets to nominal income, r_t^{cd} is the nominal rate on cash and deposit holdings, and μ^{cd} captures the responsiveness of cash and deposit holdings to changes in the real rate. Our specification reflects the use of money for transaction purposes as well as store of value. The demand for cash and deposits by the Central Bank is kept exogenous.

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[8] In the baseline model and simulations, we keep the value of λ_t^{rr} set to 0. However, this requires a very large coefficient for β^{repo}. This can lead to the conclusion that monetary policy is highly effective. While monetary policy in our framework is more effective than in models with limited or no financial sector dynamics due to the mechanisms identified by Borio and Zhu (2012), the size of the coefficient in this case also reflects a simplification, assuming the factors captured by λ_t^{rr} are directly linked to monetary policy. An alternative specification is to reduce the size of β^{repo} and either exogenously or endogenously provide values for λ_t^{rr}.

The household also demands equities, while the demand for bonds is kept exogenous. The household has a low direct exposure to bonds in the underlying data. We assume a simple relationship in which the stock of equities is equal to the financial wealth not invested in other assets. The household equity stock largely represents interests in retirement and life funds.

The demand for equities by the non-financial sector, government, and the Central Bank is kept exogenous. The decision by government to hold equities is likely to be driven by discretionary policies, while the Central Bank generally does not hold equities. In the case of the non-financial sector, our assumption aims to reduce the model complexity. The purchases of equities by this sector are also likely to be a function of various strategic considerations which go beyond equity returns. The demand for bonds by the non-financial sector is also kept constant, reflecting the decreasing direct importance of the sector in the bond market.

Government does not demand bonds as an asset. The Central Bank's demand for bonds is residual demand. It is based on the flow-of-funds identity:

$$\begin{aligned} da_t^{rb,b} = SAV_t^{rb} + \sum_{fi} dl_t^{rb,fi} + PEQ_t \cdot dl_t^{rb,e} - PEQ_t \cdot da_t^{rb,e} \\ - \sum_{fib} da_t^{rb,fib} - PRC_t \cdot I_t^{rb} \end{aligned} \tag{27}$$

where *da* is the change in the stock of assets and is defined over the set of financial instruments *fi*. Considering that the Central Bank's liabilities are made mainly of cash and deposits, the identity effectively reflects open-market operations. The bank expands money supply by purchasing bonds after accounting for net savings. This identity ensures that the supply and use of funds are equal. The same identity applies to all agents and ensures stock and flow consistency.

The 'other' financial instrument is kept constant for all agents except the Central Bank. This is because we classify foreign reserves under other assets. For the Central Bank, the accumulation of other financial assets is given by the identity:

$$SA_t^{rb,oa} = EXR_t \cdot res + sa^{rb,oa} \tag{28}$$

where *sa* is the exogenous other assets of the Central Bank, excluding foreign currency reserves. The reserves are represented in foreign currency units and are fixed (res). The other assets ($SA_t^{rb,oa}$) for the Central Bank fluctuate with changes in the exchange rate or discretionary policy decisions, which change the level of reserves.

2.7 Liabilities

The demand for loans on the liability side is modelled similarly to the demand for cash and deposits.

$$SL_t^{ins,l} = \alpha^{ins,l} \cdot \left(\frac{1 + r_t^l}{1 + \pi_t}\right)^{\mu^l} \cdot PRC_t \cdot YI_t^{ins} \tag{29}$$

The function represents the demand for loans for all institutions, except the foreign sector. The parameter $\alpha^{ins,l}$ is fixed, while the demand fluctuates with changes in the real borrowing costs. The elasticity μ^l is negative. For the foreign sector, we keep the loans a fixed share of domestic GDP lagged one period and expressed in foreign currency units. The fixed share is calibrated to the base year.

$$SL_t^{row,l} = \alpha^{row,l} \frac{\cdot PRC_{t-1} \cdot GDP_{t-1}}{EXR_{t-1}} \tag{30}$$

Government is the only institution that issues bonds. This reflects the information in our underlying data. The issuance of bonds is driven by government's decision to consume, save, invest, and accumulate financial assets and liabilities. The specification reflects the flow-of-funds identity, with bonds on the liability side being the balancing item.

The issuance of equities is modelled endogenously for the financial, non-financial, and foreign sectors. The equity issuance for the financial sector varies directly with the accumulation of equities by the household sector.

$$SL_t^{fin,e} = sl^{fin,e} + SA_t^{H,e} \tag{31}$$

Our definition of equities includes *interests in retirement and life funds*, which is the main financial asset of households and sits on the financial sector balance sheet as a liability. The other equity liabilities of the financial sector are exogenous ($sl^{fin,e}$).

The non-financial sector supplies equities on demand similarly to the specification in Godley and Lavoie (2007). The set *infin* includes all agents except non-financial institutions. Changes to equity prices and dividend payments affect the demand for equities, leading to changes in the supply of equities by the non-financial sector. This ensures that the supply and demand for equities are equal.

$$SL_t^{nfin,e} = \sum_i SA_t^{i,e} - \sum_{infin} SL_t^{infin,e} \tag{32}$$

The foreign sector equity liability is also a constant share of GDP (expressed in foreign currency units), modelled similarly to the loan liability.

Cash and deposits are created by the Central Bank and the financial sector. The Central Bank expands its money supply according to Equation 33:

$$SL_t^{cb,cd} = \sigma^{cb,cd} \cdot \sum_i PRC_t \cdot YI_t^i \tag{33}$$

The stock of cash and deposit liabilities of the Central Bank grows with the total nominal income in the economy. Higher national income translates into greater transactional demand for money. We assume that the relationship is constant and captured through the coefficient $\alpha^{cb,cd}$. The financial sector accommodates the demand for cash and deposits.

The 'other' liabilities are fixed for all institutions except the foreign sector as the foreign reserves, which fall on the asset side of the Central Bank's balance sheet, and on the liability side of the foreign sector.

2.8 Interest rates

The policy rate is the repo rate set by the Central Bank. We use a Taylor Rule specification, similarly to de Jager et al. (2015), though our coefficients are of different size, and we use deviations from trend inflation rather than inflation expectations. In our framework, inflation expectations are adaptive and thus the current specification also captures the relationship between the policy rate and inflation expectations:

$$r_t^{repo} = \rho^{repo} \cdot r_{t-1}^{repo} + (1 - \rho^{repo}) \cdot \left(inf + \beta_2^{repo} \cdot (\pi_t - inf) + \beta_3^{repo} \cdot (y_{t-1}^{gap,l} - 1)\right) \tag{34}$$

where r_t^{repo} is the repo rate and $y_t^{gap,l}$ is a proxy for the output gap, which measures the capacity utilization of labour. Interest rate decisions affect all other interest rates. While we capture the traditional channels of monetary policy mechanism, our framework also has features which capture the intermediation–interest rate spread channel and the related risk-taking channel (Borio and Zhu 2012; Woodford 2010).

$$r_t^b = r_t^{repo} + dr_t^b \tag{35}$$

$$r_t^{cd} = \mu_2^{rcd} \cdot r_t^{repo} + (1 - \mu_2^{rcd}) \cdot r_t^l \tag{36}$$

$$r_t^l = r_t^{repo} + dr_t^l \tag{37}$$

The other interest rates modelled are for bonds (r_t^b), cash and deposits (r_t^{cd}), and loans (r_t^l). In each case the interest rate is a function of the repo rate. The term *dr* fluctuates, bringing the supply and demand of the respective financial instrument into equilibrium. Unlike the equity market and the cash and deposit market, the markets for loans and bonds are brought into equilibrium via the respective interest rate, which feeds into the asset demand functions described above. The term *dr* represents the interest rate spread.

A reduction in the supply of loans increases the lending spread over the repo rate and reduces demand for loans as explained above. The spread reflects risk and market power.

The adjustment in the loan market takes place through the balance sheets of the household, financial, and non-financial sectors.

The bond market operates similarly. However, in this case the changes in demand are on the financial and foreign sectors' sides. An increase in the supply of bonds requires higher bond yield to encourage agents to purchase bonds.

2.9 Income of institutions

Every institution receives factor income (*YIF*), dividends (*DVD*), interest income (*INT*), other income (*OI*), and social contributions (*SCOC*). The government also receives tax revenue.

$$PRC_t \cdot YI_t^i = \sum_f YIF_t^{i,f} + DVD_t^i + INT_t^i + OI_t^i + SCOC_t^i \tag{38}$$

In the case of government, income is equal to

$$PRC_t \cdot YI_t^{gov} = YIO_t + ts_t \cdot PQ_t \cdot QQ_t + ta_t \cdot PA_t \cdot QA_t + tm_t \cdot pwm_t \cdot EXR_t \cdot QM_t + \sum_{insd} td_t \cdot PRC_t \cdot YI_t^{insd} \tag{39}$$

where YIO_t is explained by Equation 38; government generates tax revenue from sales taxes, activity taxes, import tariffs, and direct taxes on income such as personal income tax and corporate income tax.

These identities reflect the structure of our financial social accounting matrix (SAM), which mirrors the production and distribution accounts published by the Central Bank.

The factor income received by each domestic agent is defined as:

$$YIF_t^{i,f} = shr^i \cdot (YF_t^f - EXR_t \cdot yfrow^f) \tag{40}$$

We assume that in each period the share of labour and capital income that goes to each domestic agent is fixed. We also assume that the factor income paid to the rest of the world ($yfrow^f$) is exogenous and fixed in foreign currency units. YF_t^f is simply the product of the factor return *WF* and the quantity of factors employed *QF*. All agents except the foreign sector receive capital returns, but it is only the foreign sector and the representative household that receive wages.

Dividend income is divided according to the share of equities that each agent holds. For example, the more equities the household holds, the more dividend income it will receive. There are three sources of dividend income. These are the financial and non-financial sectors and the rest of the world. The dividend income from the foreign sector is exogenous and fixed in foreign currency units. The dividend payments by the financial and non-financial sectors are determined by Equation 41:

$$DVDP_t^{iii} = (1 - td_t^{iii}) \cdot PRC_t \cdot YI_t^{iii} - SC_t^{iii} - OIP_t^{iii} - IEXP_t^{iii} - SAV_t^{iii} \tag{41}$$

Dividends paid by the financial and non-financial sectors (the set *iii*) are a function of after-tax income, social contributions, interest expenditure, and other expenditure paid, as well as the savings decisions of the two sectors. It is important to note that dividend payments can be negative, which is equivalent to the holders of equity injecting money into the two sectors. Higher savings reflect higher retained earnings.

Interest income is divided similarly to dividend income. All the interest payments go into a pool, which is divided according to the holding of interest-bearing assets by the various agents. The interest paid by each agent is defined in Equations 42 and 43:

$$IEXP_t^i = r_{t-1}^l \cdot SL_{t-1}^{i,l} + r_{t-1}^{cd} \cdot SL_{t-1}^{i,cd} + IEXPB_t^i \tag{42}$$

$$IEXPB_t^i = r_{t-1}^b \cdot dl_{t-1}^{i,b} + IEXPB_{t-1}^i \tag{43}$$

Changes to the interest rates of loans and cash and deposits apply to the entire liability stock, whereas for bonds the change applies only to debt issued in the same period. $IEXPB_t^i$ is the interest income generated on bonds.

The other income received by agents is a fixed share of the pool of other payments. We assume that the share does not change. For domestic agents, we assume that the other payments are a fixed share of GDP in the previous period, while for the foreign sector they are fixed in foreign currency units and fluctuate with changes in the exchange rate. The social contribution income is modelled the same way.

2.10 System constraints

Our system is stock and flow consistent. The model system constraints apply to both the real and the financial sides. The real-side constraints are similar to those in other CGE models such as that developed by Devarajan and Go (1998). The income and expenditure must be equal. In addition to the real balances, in a financial SAM we have to add the financial balances. The financial SAM enforces flow consistency across real and financial flows. The sources of funds must equal the uses of funds for every institution, and the total change (across all institutions) in the holding of a financial instrument on the asset side must be equal to the change on the liability side.[9]

The first real-economy constraint is that the total supply must be equal to the total demand in the economy.

$$QQ_t = QINTA_t + QH_t + QG_t + \sum_i INV_t^i + inv \tag{44}$$

The term *inv* represents the change in inventories, which are exogenous in our framework. The supply in the economy is given by imports and the domestically produced good supplied to the local market.

In our framework, the demand for factors of production (Equation 45) is a function of y_t^{gap}(Equation 46), which is a proxy for the output gap. The y_t^{gap} can vary between 0.95 and 1.05, indicating that demand for labour and capital can be slightly below or above the supply (i.e. a negative or positive output gap). It captures deviations from full employment levels driven by the economic cycle. While this is a hard constraint, the structure of the model does not allow for it to become binding.[10] The response of Equations 25 (reserve ratio), 34

[9] This also represents the approach followed in the compilation of flow-of-funds data, including the South African flow-of-funds information as published by the South African Reserve Bank.

[10] During the calibration process, the imposition of the constraint led to an effective way of ensuring that the system properties of never generating output gaps larger than 5% are achieved. The initial imposition of the

(Taylor Rule), and 46 (capacity utilization), and the responses in the demand for goods, services, and financial assets and liabilities, ensure that deviations from potential growth are rapidly corrected.

$$QF_t^f = y_t^{gap,f} \cdot QFS_t^f \tag{45}$$

The proxy variable is a function of the real loan rate and deviations of aggregate demand from its steady state. An increase in the real rate reduces the demand for factors of production (the output gap becomes more negative), whereas aggregate demand growth which exceeds the steady-state growth rate increases the demand for factors of production.

$$y_t^{gap,f} = y_{t-1}^{gap,f} - \alpha_1^{gap,f} \cdot \left(r_{t-1}^l - \pi_{t-1}\right) + \alpha_2^{gap,f} \cdot (\Delta AD_{t-1} - \Delta ad) \tag{46}$$

The elasticities $\alpha_1^{gap,f}$ and $\alpha_2^{gap,f}$ reflect the responsiveness of the proxy measure to the change in the real rate and the deviation of aggregate demand from its steady-state growth level. The coefficients for labour are larger, indicating that labour changes are more sensitive to changes in aggregate demand and the interest rate cycle. The specification also implies that factor returns adjust. A fall in factor costs indicates that either the supply is rising faster than the demand or y_t^{gap} is falling.

The next constraint is the current-account balance constraint, expressed in foreign currency units:

$$SAVF_t = pwm_t \cdot QM_t - pwe_t \cdot QE_t - TRANSF_t \tag{47}$$

where $TRANSF_t = nyif + NDVD_t + NOI_t + NINT_t$.

Foreign savings (*SAVF*) are equal to the trade balance minus the balance on the income portion (*TRANSF*) of the current account. The latter is a function of net factor payments ($nyif$), which are fixed and exogenous, net dividend receipts (*NDVD*), net interest receipts (*NINT*), and net other income (*NOI*). We assume that foreign savings are fixed, which implies that the current-account deficit is fixed. The exchange rate adjusts to ensure that the equilibrium is maintained. This is our closure with respect to the external balances. For example, an increase in dividend outflows, holding foreign savings fixed, requires the exchange rate to depreciate, reducing the outflows in foreign currency units and imports, while increasing exports.

The last real-economy constraint is the savings–investment balance, which ensures that total savings in the economy are equal to total investment. The adjustment, as indicated earlier, takes place through the savings rate of the financial and non-financial sectors.

constraint led to the model generating infeasible solutions. By changing key coefficients (in Equations 25, 34, and 46, and the demands for loans and cash and deposits), the model property of oscillation within a range of negative output gap of 5% to positive output gap of 5% was achieved. The approach to calibrating all parameters is explained in Section 3.4.

$$\sum_i SAV_i^i + EXR_t \cdot SAVF_t = \sum_i PRC_t \cdot INV_i^i + inv \tag{48}$$

There are two financial constraints. The first is that the sources of funds must be equal to the uses of funds for every institution. This constraint is enforced though our approach of calculating gross financial wealth available for investment in every period and its allocation to various assets. The second constraint is that the stock of liabilities is equal to the stock of financial assets for every financial instrument.

$$\sum_i SA_t^{fi,i} = \sum_i SL_t^{fi,i} \tag{49}$$

For loans and bonds, this is achieved through the loan and bond rates respectively. For equities and cash and deposits, the adjustment takes place through the non-financial equity liabilities and the cash and deposit liability of the financial sector. In the case of other assets, the adjustment takes place through the other liabilities of the foreign sector.

2.11 Consumers

The entire system of equations is solved maximizing the household utility function:

$$U_0 = \sum_{t=0}^{10} \beta^t \, Log(QH_t) \tag{50}$$

where $\beta = (1+\rho)^{-1}$, ρ is a positive parameter, and β is the implied discount factor. In every period the household solves an inter-temporal optimization problem to determine the current value of its consumption and savings, based on the real financial wealth that it wants to achieve in ten periods, plus the model constraints determined by the other equations in the system. The choice of a finite period reflects the permanent income hypothesis. Households base their consumption not on their current income, but on the income they generate over a period of time. The target real net wealth is set as follows:

$$FW_{t,t+10}^H = FW_{t-1}^H \cdot (1+\mu^{fw})^{10} \tag{51}$$

The target wealth in the current period is based on the solution from the previous period adjusted for a real return μ^{fw}.

2.12 Dynamics

The dynamics of the model presented here differ from those of standard recursive CGE models. On the one side, the inter-temporal optimization of the household requires that the model system is solved simultaneously over a finite period of time. At the same time, once a solution is found, the variables in the current period become fixed-state variables for the inter-temporal optimization in the next period. Figure 2 explains the solution process. The boxes represent the optimal path. In principle, the model could run forever; however, we limit the solution to 22 periods. At period *t*, the representative household decides on consumption and savings based on its expectations about the economy over the next ten periods and a level of wealth that it wants to achieve. The target is based on an exogenous growth rate, which indicates the real wealth that the representative household would like to achieve in the future given its current level of wealth. For simplicity, the wealth target assumption is adopted in the tradition of Pigou's real wealth effect (Patinkin 1948; Tobin

1975). Once a solution is found for period *t*, the household solves for period *t+1*. The solution values for period *t* are used as starting values for period *t+1*.[11]

Our assumptions about expectations are different from those of mainstream DSGE models. The household has model-consistent expectations (similarly to DSGE models) within each period. It has good understanding of the structure of the economy and uses the rules in the model to form expectations. However, the ability of the representative household to foresee the future is limited to ten periods (two-and-a-half years)[12] and the formation of expectations can vary between periods. Newly formed expectations can be introduced, for example, by shortening or increasing the optimization period, or changing the value of coefficients or the structure of equations between periods. As the household solves for each period, new information about the economy becomes available, which is incorporated into the next period's optimization. Our expectation formation resembles the process identified by Roos and Luhan (2013). They find that households have bounded rationality rather than full rationality.[13] A significant number of households use more sophisticated models of the economy, taking additional information into account as it becomes available to form expectations, but they do not have perfect foresight. The presence of only one household in our framework, however, limits our ability to capture heterogeneity of expectations.

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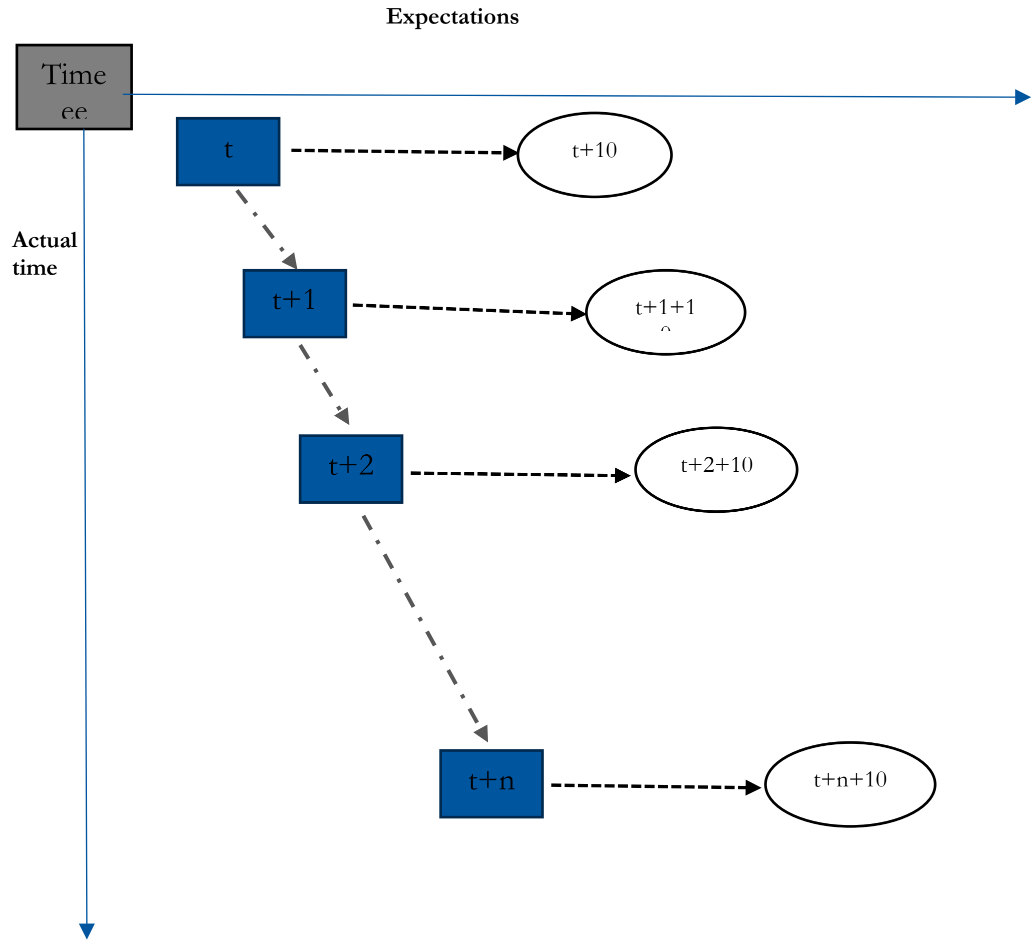
[11] In terms of model dynamics, the solution process is introduced through a loop, which traces the path identified by the blue boxes in Figure 2. Once a solution is found for *t*, the loop moves to the next period. In each case the solution for the variable represented by the blue box is derived through inter-temporal optimization reflecting the structure of the model economy and the household desire to achieve a certain level of wealth in the future.

[12] The assumption of ten periods reflects the period that monetary shocks take to dissipate in an economy and the inflation expectations period generally targeted by central banks. We have assumed that this period also reflects the household expectation horizon.

[13] Hommes (2011) provides a review of the literature on bounded rationality. The theory of bounded rationality originates in the seminal work of Simon (1955).

Figure 2: The household optimization path



Source: Authors' elaboration.

The model description above has outlined some of the dynamic equations. These include, for example, prices (Equation 11), price expectations (Equation 16), financial wealth (Equation 22), the Tobin asset demand function (Equation 26), and the Taylor Rule (Equation 34). In addition to these, the capital stock follows the standard approach, in which capital stock (QFS_t^k) in the current period is a function of capital stock in the previous period after depreciation (δ) plus total real investment in the previous period.

$$QFS_t^k = (1-\delta)\cdot QFS_{t-1}^k + \sum_i INV_{i,t-1} + inv \tag{52}$$

The labour supply (QFS_t^l) is simply modelled as growing by an exogenous growth rate ρ^{lab}.

$$QFS_t^l = (1+\rho^{lab})\cdot QFS_{t-1}^l \tag{53}$$

Finally, the household optimization is assumed to generate a constant savings rate (savings plan) over the ten-quarter optimization period. This is driven by Equation 54, which ensures that the after-tax savings rate in the current period is equal to the rate in the previous period.

$$\frac{SAV_t^H}{(1 - td_t^H) \cdot PRC_t \cdot YI_t^H} = \frac{SAV_{t-1}^H}{(1 - td_{t-1}^H) \cdot PRC_{t-1} \cdot YI_{t-1}^H} \quad (54)$$

The solution process for households is different to that of conventional models, which use savings to smooth consumption. In our model, wealth accumulation is more important than consumption. Savings-smoothing ensures a more stable path for household wealth as well as for interest and dividend income.

Demand arises from the household, government, investment, and net exports. The growth rate in aggregate demand reflects: the household's decision to accumulate wealth given its total income and the performance of financial markets; government's exogenous growth rate in consumption; investment by institutions, which is a function of income, borrowing costs, and equity prices (in the case of non-financial firms); and relative export and import prices, domestic demand, and production, which determine net exports. The growth rate in aggregate demand, which exceeds the steady state Δad, pushes $y_t^{gap,f}$, which leads to higher borrowing costs in the economy and starts a process of re-equilibrating.

3 Data

In this section, we present the data used to calibrate the model. We construct financial macro SAMs for the South African economy over the period 2001 to 2012. Our approach follows the method outlined by Emini and Fofack (2003) and Hubic (2012). Capital and financial blocks are added to the standard SAM. These reflect the transactions that take place in the financial sector: the incurrence of liabilities and the accumulation of assets by institutions. The changes in liabilities and assets for a particular institution also reflect how the savings–investment balance (capital account) is financed.

Table 1: Aggregation of financial instruments

Cash and deposits	**Equities**	**Loans**
Cash and demand monetary deposits Short-/medium-term monetary deposits Long-term monetary deposits Deposits with other financial institutions Deposits with other institutions	Securities of public enterprises Ordinary shares Interest in retirement and life funds Other loan stock and preference shares	Bank loans and advances Trade credit and short-term loans Long-term loans Mortgage loans
Bonds	**Other**	
Treasury bills Short-term government bonds Long-term government bonds Non-marketable government bonds Securities of local governments	Gold and other foreign reserves Other bills Foreign branch/head office balances Amounts receivable/payable Other assets/liabilities Balancing item	

Source: Authors' own calculation based on South African Reserve Bank data.

The financial block requires flow-of-funds data, which is produced by the South African Reserve Bank (SARB) and is available from 1970. The South African data provides for 11 institutional units and 23 financial instruments. These are aggregated into six institutions and five financial instruments. The aggregation is driven by: availability of production and distribution accounts, which are available on a more aggregate level; other data limitations, which we outline below; and the need to reduce the computational and behavioural complexity of the model. After the aggregation, the financial instruments are cash and deposits, equities, bonds, loans, and other, while the institutions are the representative household, the financial sector, the Reserve Bank, the representative non-financial firm, government, and the foreign sector.

The aggregation of institutions does not lead to consolidation of flows—i.e flows between institutions which are part of the same category are not netted out. This reflects the absence of whom-to-whom accounts in South Africa as well as the presence of the same practice in the production, accumulation, and distribution accounts.

Table 1 and Table 2 summarize the aggregation. The financial macro SAMs impose flow consistency on the model.

Table 2: Institutional aggregation

Foreign sector	SARB	Financial sector
Foreign sector	Monetary authority	Other monetary institutions Public investment corporation Insurers and retirement funds Other financial institutions
Government	**Non-financial enterprises**	**Households**
Central and provincial government Local government	Public sector Private sector	Households

Source: Authors' own calculation based on South African Reserve Bank data.

The stock consistency requires the construction of balance sheets which are consistent with the flow-of-funds data. While the Reserve Bank provides balance sheet information for some institutions, it is impossible to link the balance sheet information to the flow-of-funds data as there is no consistency in terms of the financial instruments listed in the different tables. The quality of balance sheet information is a significant limitation of the framework that we present. This is a global problem which is highlighted by the G20 Data Gap Initiative.[14]

We provide a short overview on our approach to constructing financial balance sheets. The building of balance sheets for institutions relies on flow-of-funds data from 1970 onwards and the balance sheet information available in the *Quarterly Bulletin* published by the South African Reserve Bank. It is important to note that our institutional balance sheets deal only with financial instruments, as consistent data on ownership of non-financial assets and liabilities is not available. Thus, our balance sheets are partial but consistent when it comes to financial assets and liabilities.

[14] For more information on the G20 Data Gap Initiative see FSB (2016).

The general approach followed is to add up the data from the flow of funds over the period 1970 to 2001. This calculates the stock of assets and liabilities accumulated over the period. While the values are smaller than the actual stock of assets and liabilities, they will be a good approximation, as the values over the period 1970 to 2001 should exceed by far the stock values prior to 1970. They should constitute most assets and liabilities accumulated. The approach is similar to the perpetual inventory method and resembles the method employed by Aron and Muellbauer (2006) and Aron et al. (2006). They estimate household wealth and balance sheets for South Africa.

Our second step is to compare the values of stocks generated in the first step against the balance sheet information presented in the *Quarterly Bulletin* for some of the institutions. Generally, the approach using the flow-of-funds data generates stock variables, which are in line with the *Quarterly Bulletin* data for deposits and some of the loan variables, but significantly different for equities and bonds. This is due to the flow-of-funds data recording some transactions at book value and some at market value, creating a discrepancy between the balance sheets generated in the first step and the balance sheets presented in the *Quarterly Bulletin*.

A key challenge with the flow-of-funds data is that they do not distinguish between changes due to changes in the holding of units, changes due to changes in the price of the financial instrument, and other changes. The second challenge is that the financial instruments in the flow-of-funds data are not directly comparable with the financial instruments used in the different balance sheets presented in the *Quarterly Bulletin*. More importantly, balance sheets are not produced for all institutions and there is no consistency in terms of the representation of financial instruments. This requires that we create our own consistent financial balance sheets.

In the next step, the data are updated with the balance sheet information from the *Quarterly Bulletin* where significant differences exist and it is reasonable to assume that the differences are primarily because of uncaptured capital gain effects. The flow-of-funds data are applied to the updated balance sheet information for 2001 to generate balance sheet information for the period 2002 to 2012. This approach guarantees that the balance sheet data generated over the period 2002 to 2012 links to the set of financial SAMs through the changes in assets and liabilities. The approach ensures consistency in stocks and flows. The data are aggregated following the matching in Table 1 and Table 2. The stock variables are not consolidated during the aggregation process—i.e. no netting out takes place. This is driven again by the absence of whom-to-whom accounts and the use of the same practice in the production, accumulation, and distribution accounts.

The balance sheet data from the *Quarterly Bulletin* is also compared with the balance sheets calculated using the flow of funds for 2012. Again, there are some discrepancies for some financial instruments, particularly equities. Despite these differences, no further changes are made to the balance sheet data calculated using the flow-of-funds data, as this requires changes to the underlying net purchases of financial assets and liabilities. This will break the link with the financial SAMs and the savings and investment data, and create flow and stock inconsistencies in our data set.

The absence of separate price and quantity effects in the flow-of-funds data hinders the modelling of prices for financial instruments, particularly the prices of bonds and equities. We model only the equity price, which is based on the Johannesburg Stock Exchange All Share Index. The adjustment to the equity stocks follows the same approach as outlined by

Aron and Muellbauer (2006). This adjustment leads to equity stock values which are more in line with the balance sheet information from the *Quarterly Bulletin*.

The two categories *Amounts receivable/payable* and *Other assets/liabilities* are grouped in a category labelled *Other*. These two items tend to generate large negative flows for some institutions, which persist over a long period of time. The behaviour of these flows reflects the current challenges with the flow-of-funds data, which are expected to be resolved as part of the G20 Data Gap Initiative. We have chosen to treat the *Other* category as a residual item, which is important in order to balance the stocks and the financial macro SAMs, but it has no behavioural function in our model.

Table 3 and Table 4 provide a summary of the balance sheets generated through the process described above as well the assets and liabilities owned by the various institutions. While our aim is not to provide a detailed assessment of the balance sheet information, we need to highlight some trends.[15]

The largest compositional changes are experienced by the Reserve Bank and the financial, non-financial, and foreign sectors. For the Reserve Bank, the share of loans as part of assets and liabilities has declined while the share of cash and deposit liabilities has increased over the period 2001 to 2012. In the case of the financial and non-financial sectors, there has been a notable increase in holding of equities as a percentage of total assets and liabilities. This reflects price effects but also greater issuance of shares. The share of loans as assets on the balance sheet of non-financial institutions has been halved. The share of loans on the balance sheet of financial institutions is small—around 20 per cent. This reflects our aggregation approach, which groups other monetary institutions with all other financial institutions, thus diluting the share of total loans in overall assets. The share of equities has significantly increased in the foreign portfolio of residents. This reflects the relaxation of exchange controls, which has led to the increased purchase of equities by South African residents.

Looking at the specific financial instruments in Table 4, the value of equities has seen the strongest growth over the period, largely driven by a significant increase in the price.

The financial sector has the largest share of equities on the asset and liability sides compared with other institutions. On the liability side, the share reflects our classification approach. Household *Interests in insurers and retirement funds* were classified as part of the financial sector equity liability. This also explains the large equity ownership share of households. Household financial wealth is mainly in the form of *Interests in retirement and life funds*, which accounted for almost 90 per cent of total household financial assets in 2012. This reflects the South African pension system, where individuals' contributions to retirement annuities and pension funds during their working life are used to purchase pension upon retirement.

The financial sector, as expected, holds almost all the cash and deposits in the economy on the liability side (92 per cent in 2012), in line with its intermediation function. The sector also holds a large share of the cash and deposits assets, followed by the non-financial sector and households.

The largest holders of government bonds are the financial sector, with close to 77 per cent of all bonds, a share which has been stable over the period 2001 to 2012, and the foreign sector, whose share has doubled over the period. The foreign sector's importance as funder

[15] For more information on South Africa's flow-of-funds data see Monyela and Madonsela (2015).

of the domestic economy has increased significantly, with the asset side increasing by more than 700 per cent. This reflects relaxation of capital controls, high global liquidity, and low domestic savings. The disproportional increase in the liability side implies that depending on the rates of return on the asset and liability side and the exchange rate, the net dividend and interest income outflows are likely to remain large and contribute negatively to the current account.

Loans are mainly provided by financial institutions, but also by non-financial enterprises and the foreign sector. Non-financial enterprises provide mainly trade loans and are also a large recipient of loans, along with households. For households, close to 50 per cent of their loan liability is in the form of *mortgages*, while for non-financial institution the loan category is dominated by *bank loans and advances*.

The *other* category is negative for some institutions. This reflects the challenges with the flow-of-funds data, which were explained earlier. The *other* category is used as an exogenous item. It is modelled endogenously only for the Reserve Bank and the foreign sector. Foreign reserves of the Reserve Bank are classified in the *other* category. They are an asset for the Bank and a liability for the foreign sector. We described our modelling approach to the foreign currency reserves in the previous section.

The data used for the econometric calibration of some of the coefficients is sourced from the Reserve Bank and is also published in the *Quarterly Bulletin*. This is data on interest rates, growth rates, and price indices.

Table 3: Composition of partial financial balance sheets of institutions

Year 2001 (% of total)	Reserve Bank		Financial institutions		Governments		Non-financial institutions		Households		Rest of the world	
	Assets	Liabilities	Assets	Liabilities	Assets	Liabilities	Assets	Liabilities	Assets	Liabilities	Assets	Liabilities
Cash and deposits	0.1	43.7	14.0	36.5	48.9	0.0	34.7	1.2	26.3	0.1	10.5	13.7
Bonds	6.2	0.0	9.9	0.0	2.3	75.8	5.9	0.0	0.1	0.0	6.7	0.3
Equities	1.6	6.9	37.7	51.7	4.2	0.3	12.7	72.0	78.5	0.0	74.5	41.4
Loans	19.1	53.1	26.8	4.1	35.2	13.7	39.3	30.6	1.8	83.1	11.9	28.7
Other	73.0	−3.7	11.6	7.7	9.4	10.1	7.5	−3.8	−6.7	16.8	−3.6	15.8
Total (R million)	**148 908**	**110 755**	**2 867 644**	**2 866 737**	**108 290**	**516 141**	**1 071 964**	**1 083 222**	**1 170 652**	**718 843**	**468 229**	**539 989**
Year 2012 (% of total)												
Cash and deposits	4.2	75.8	16.1	29.4	99.9	0.0	31.4	0.1	13.7	0.0	2.4	0.2
Bonds	1.2	0.0	6.9	0.0	0.5	70.2	0.9	0.0	0.5	0.0	5.4	0.0
Equities	2.0	5.0	52.0	62.7	10.2	0.7	30.3	83.4	87.6	0.0	70.6	74.7
Loans	5.5	32.5	21.3	3.8	44.8	8.0	19.7	22.1	0.5	77.8	13.9	10.0
Other	87.1	−13.3	3.7	4.1	−55.4	21.1	17.6	−5.6	−2.2	22.2	7.7	15.2
Total (R million)	**502 472**	**462 685**	**13 296 904**	**14 603 042**	**504 817**	**1 698 866**	**3 044 478**	**6 879 223**	**6 858 709**	**2 141 453**	**3 862 913**	**2 285 463**
% change	*237.4*	*317.8*	*363.7*	*409.4*	*366.2*	*229.1*	*184.0*	*535.1*	*485.9*	*197.9*	*725.0*	*323.2*

Source: Authors' own calculation based on South African Reserve Bank data.

Table 4: Ownership of financial instruments

Year 2001 (% of total)	Equities		Cash and deposits		Loans		Bonds		Other	
	Assets	Liabilities	Assets	Liabilities	Assets	Liabilities	Assets	Liabilities	Assets	Liabilities
Reserve Bank	0.1	0.3	0.0	6.3	2.1	4.4	2.3	0.0	25.0	−1.0
Financial institutions	43.4	59.4	34.0	88.4	57.6	8.8	72.6	0.0	76.3	51.0
Government	0.2	0.1	4.5	0.0	2.9	5.3	0.6	99.6	2.3	12.0
Non-financial institutions	5.5	31.3	31.4	1.1	31.6	24.9	16.1	0.0	18.4	−9.5
Households	36.9	0.0	26.0	0.1	1.6	44.9	0.4	0.0	−18.1	27.7
Rest of the world	14.0	9.0	4.1	6.3	4.2	11.6	8.0	0.4	−3.8	19.6
Total (R million)	**2 493 277**	**2 493 277**	**1 183 143**	**1 183 143**	**1 331 370**	**1 331 370**	**392 842**	**392 842**	**435 055**	**435 055**
Year 2012 (% of total)										
Reserve Bank	0.1	0.1	0.5	7.5	0.6	3.5	0.5	0.0	22.4	−4.6
Financial institutions	41.5	55.0	46.1	92.2	66.5	13.0	76.9	0.0	37.0	45.1
Government	0.3	0.1	10.8	0.0	5.3	3.2	0.2	100.0	−20.9	26.9
Non-financial institutions	5.6	34.5	20.5	0.2	14.1	35.7	2.2	0.0	40.2	−28.9
Households	36.1	0.0	20.1	0.0	0.7	39.2	2.8	0.0	−11.4	35.6
Rest of the world	16.4	10.3	2.0	0.1	12.6	5.3	17.4	0.0	22.4	26.0
Total (R million)	**16 629 425**	**16 629 425**	**4 659 809**	**4 659 809**	**4 253 065**	**4 253 065**	**1 192 420**	**1 192 420**	**1 335 574**	**1 335 574**
% change	*567.0*	*567.0*	*293.9*	*293.9*	*219.5*	*219.5*	*203.5*	*203.5*	*207.0*	*207.0*

Source: Authors' own calculation based on South African Reserve Bank data.

4 Calibration

In this section, we explain our calibration strategy and present the value of the coefficients.

The data used is a series of financial SAMs over the period 2002 to 2012 and National Accounts data for South Africa. The data sources are discussed in the previous section.

The derivation of the scale and share parameters for the Armington and CET functions follows the standard approach in CGE models as described by Condon et al. (1987). We use 2002 as the base year, but we also compare the values of coefficients and ratios using later years. There are several coefficients, which are calibrated directly from the base year macro SAM. These include tax rates, various share parameters, and the relationship between intermediate inputs and value added. Prices such as composite supply price (PQ), equity price (PEQ), and the exchange rate are set to 1 in the first quarter of 2002. The financial balance sheets for 2002 are used with the base year SAM to generate coefficients which link economic behaviour to the holding of stocks. For example, these include the parameters $\alpha^{ins,l}$ (in the loan demand function) and $\alpha^{il,cd}$ (in the cash and deposit function). The values of all coefficients are listed in Appendix B.

The substitution elasticities are based on the recent analyses by Kreuser et al. (2015) and Saikkonen (2015). The two studies provide substitution elasticities for several sectors. Our factor substitution elasticity is in the lower range of those provided by Kreuser et al. (2015), in line with the short-term nature of our framework. Our Armington elasticity is 0.5, while our CET elasticity is 0.2. Saikkonen (2015) finds that the Armington elasticities vary between 0.39 and 1.38. Hence, we assume a low CET elasticity consistent with available evidence on limited export response to exchange rate shifts.

A key challenge with our model is that many of the coefficients related to financial behaviour are not available for South Africa. This is a large area for future research. Our strategy here is to utilize coefficients generated by other studies, bearing in mind the limitations of this approach, or to get some sense of the relationship through simple econometric estimates, which are further calibrated in the model to generate a consistent baseline.

The coefficients for the asset demand function are based on those used by Godley (1996) and Godley and Lavoie (2007). The coefficients reflect the stronger response of equity and bonds to changes in relative prices. The λ_{i0} coefficients, which reflect the initial shares, are calibrated using our balance sheet data for 2002.

An econometric approach is used to generate priors for several elasticities. These include the elasticities in the equation for the reserve ratio, the elasticities μ^{l} and μ^{cd} in the demand for loans and cash and deposits equation, and $\alpha_1^{gap,f}$ and $\alpha_2^{gap,f}$ in the equation defining the output gap specification. The priors are manually adjusted so that the model generates a consistent baseline.

Steady-state growth rates reflect average values for the period 2002 to 2012, using quarterly annualized data. The growth of the labour force represents the average quarterly labour force growth in South Africa. Growth in government consumption expenditure and total factor productivity is calibrated similarly. The inflation target is assumed to be 6 per cent, in line with the upper bound of the inflation target and the level of inflation expectations of trade unions and businesses, which appear to be stuck at that level.

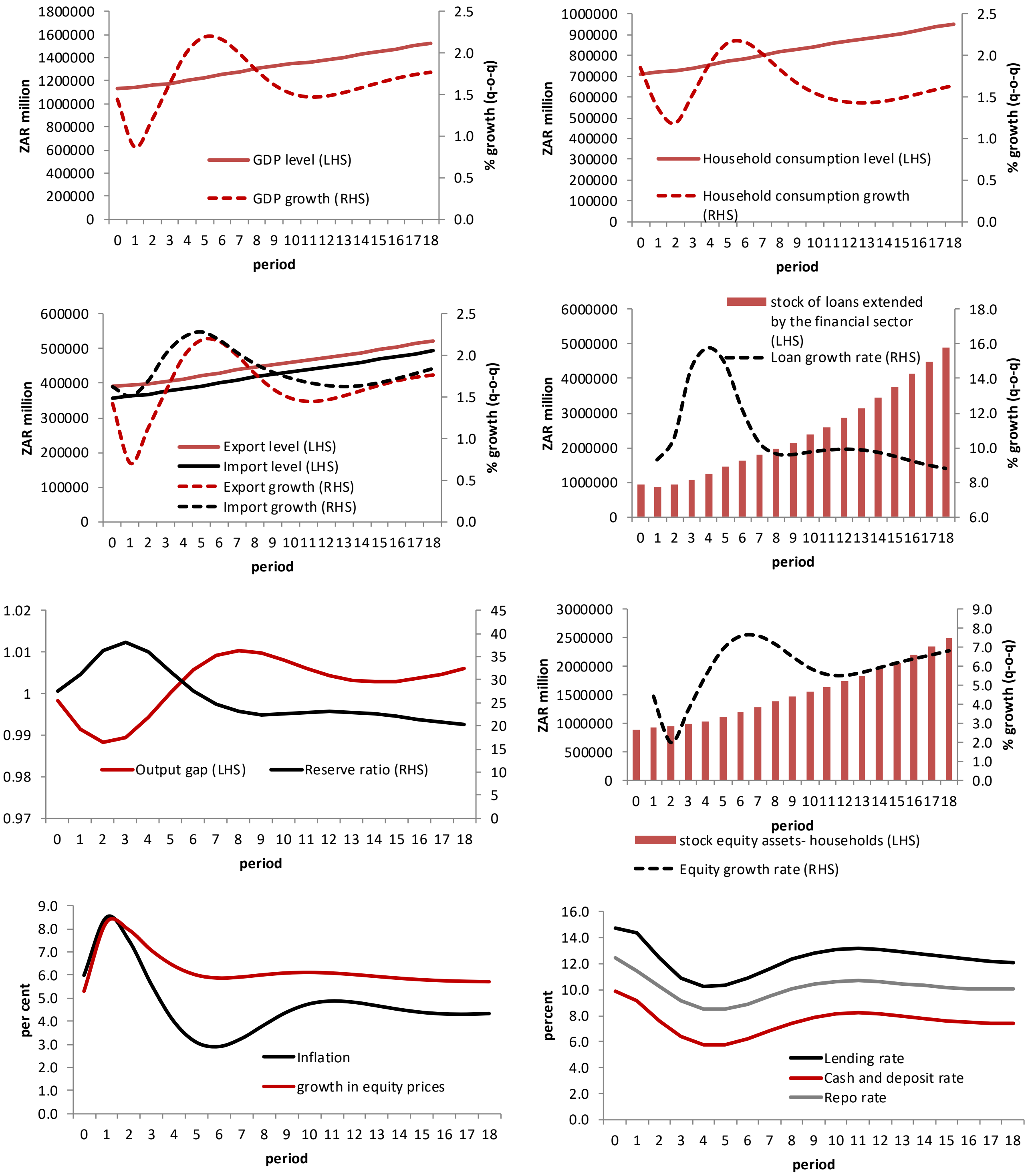
Finally, several exogenous variables are fixed to the baseline year. These include foreign savings, income generated by non-residents, foreign income generated by residents, interest income on the liabilities of the foreign sector, the bond and cash and deposit liabilities of the foreign sector, and foreign currency prices of South African exports and imports. Exogenizing these variables and fixing them to the base year values reduces the complexity of the model and eliminates the shocks that may come from their changing values.

5 Model baseline

In Figure 3, we present the model-generated baseline, given the choice of values for the coefficients and the set of exogenous variables. The set of initial conditions implies that inflation is rising at the beginning of the baseline period. This causes the repo rate, through the Taylor specification in Equation 34, to increase, which in turn affects the lending rate. The higher rates reduce investment and consumption growth, which slows down overall GDP growth. The slowdown in the economy affects the willingness of banks to hold reserves through Equation 25. The reserve ratio initially increases because of the higher repo rate. As the slowdown in economic activity also affects the growth in financial sector assets, this increases the willingness of the sector to hold reserves and puts additional pressure on the reserve ratio. The economic cycle affects probabilities of default, valuations, and the perception of risk, which changes the financial sector's willingness to hold reserves. The slower pace of economic activity also affects the household creation of equity assets. As the economy slows down and inflationary pressures decrease, the policy rates, along with other interest rates, start to fall. This provides support to aggregate demand, which accelerates. With greater economic activity, financial activity also accelerates, supported by the falling willingness of the financial sector to hold reserves and lower lending spreads. Stronger economic activity and growth in money supply support equity growth through Equation 17, which supports wealth creation. The growth in loans is in line with GDP growth and falling lending rates. As the loan rate starts to rise, lending slows down and the GDP growth rate moderates.

The quarterly growth rates for aggregate demand variables are on average in the region of 1.5 to 2 per cent higher than the actual growth rates achieved. This reflects the fact that some of the key variables are kept fixed and exogenous. The model achieves stability in the output gap and the reserve ratio to the extent that these as well other variables tend oscillate around a trend path. This represents the steady state of the model. The system is driven by a mechanism that ensures that the model framework always tries to converge to an output gap of zero. It always oscillates around an output gap of zero, similarly to real economies. The steady-state dynamics are different to those of current DSGE models, which return to constant steady-state growth rates. A shock to the system shifts the cycle. The steady state is achieved through a set of dynamic equations and parameters, which represent steady-state growth rates. The key equations are 25 (reserve ratio), 34 (Taylor Rule), and 46 (capacity utilization). Parameters include the steady-state growth rate in aggregate demand Δad, government consumption expenditure grg_t, labour force growth rate ρ^{lab}, and the steady-state growth in equity prices ss^{peq}. The sets of equations and parameters ensure that deviation from potential growth is corrected and capital and labour grow at similar rates in the baseline, keeping relative prices constant.

Figure 3: Baseline path



Note: LHS = left-hand scale; RHS = right-hand scale.

Source: Authors' elaboration based on model simulation.

6 Conclusion

The model presented in this paper captures financial sector dynamics by introducing stock and flow consistency in real and financial flows and incorporating elements of the theoretical models developed by Borio and Zhu (2012) and Woodford (2010). This provides for richer representation of financial sector dynamics compared with current DSGE and other stock and flow models.

As with any model, our framework is a simplified version of reality and is subject to limitations. The first one is our ability to capture heterogeneity among agents in the same sector. For example, we have only one household. The implications of this are that we cannot capture all the elements of risk-taking as identified by Borio and Zhu (2012) and our framework, while building on current DSGE models, is also subject to some of the same criticisms. The time profile of financial instruments is also missing from our framework. For example, we do not distinguish between short- and long-term bond instruments.

Heterogeneity can be introduced, for example, by distinguishing between credit constraint and non-credit constraint households. However, heterogeneity also introduces complexity, which can hinder our ability to trace economic shocks through the model.

The challenges with data quality and the lack of economic research analysing some of the relationships that we model impose limitations on our model. As indicated already, the flow-of-funds data and the available balance sheet data for South Africa require some improvements, which are currently being implemented as part of South Africa's commitments to the G20 Data Gap Initiative. This should provide for better quality and availability of data. Improvements to the data will also allow for the use of more advanced techniques in calibrating the coefficients, such as the maximum entropy approach.[16] More micro research into understanding bank behaviour and the interactions between the balance sheets of different institutions in South Africa would greatly enhance the robustness of our framework and the results.

[16] See for example Arndt et al. (2002). They use the maximum entropy approach to estimate the parameters of a CGE model for Mozambique.

References

Amano, R., J. Engle-Warnick, and M. Shukayev (2011). 'Price-Level Targeting and Inflation Expectations: Experimental Evidence'. Bank of Canada Working Papers Series 2011/18. Ottowa: Bank of Canada.

Armington, P.S. (1969). 'A Theory of Demand for Products Distinguished by Place of Production'. *Staff Papers (International Monetary Fund)*, 16(1): 159–78.

Arndt, C., S. Robinson, and F. Tarp (2002). 'Parameter Estimation for a Computable General Equilibrium Model: A Maximum Entropy Approach'. *Economic Modelling*, 19: 375–98.

Aron, J., and J. Muellbauer (2006). 'Estimates of Household Sector Wealth for South Africa, 1970–2003'. *Review of Income and Wealth*, 52: 285–307.

Aron, J., J. Muellbauer, and J. Prinsloo (2006). 'Estimating the Balance Sheet of the Personal Sector in an Emerging Market Country: South Africa 1975–2003'. Working Paper 2006/99. Helsinki: UNU-WIDER.

Backus, D., W. Brainard, G. Smith, and J. Tobin (1980). 'A Model of U.S. Financial and Nonfinancial Economic Behavior'. *Journal of Money, Credit, and Banking*, 12: 259–93.

Barwell, R., and O. Burrows (2014). 'Growing Fragilities? Balance Sheets in the Great Moderation'. In B. Winkler, A. van Riet, and P. Bull (eds), *A Flow-of-Funds Perspective on the Financial Crisis. Volume II: Macroeconomic Imbalances and Risks to Financial Stability*. London: Palgrave Macmillan.

Bernanke, B.S., M. Gertler, and S. Gilchrist (1999). 'The Financial Accelerator in a Quantitative Business Cycle Framework'. In J.B. Taylor and M. Woodford (eds), *Handbook of macroeconomics. Volume 1C*. Amsterdam, New York, and Oxford: Elsevier Science, North-Holland.

Blanchard, O. (2016). 'Do DSGE Models Have a Future?' PIIE Policy Brief 16-11. Washington, DC: Peterson Institute for International Economics.

Borio, C., and H. Zhu (2012). 'Capital Regulation, Risk-Taking and Monetary Policy: A Missing Link in the Transmission Mechanism?' *Journal of Financial Stability*, 8: 236–51.

Burgess, S., O. Burrows, A. Godin, S. Kinsella, and S. Millard (2016). 'A Dynamic Model of Financial Balances for the United Kingdom'. Bank of England Staff Working Paper 614. London: Bank of England.

Caballero, R.J. (2010). 'Macroeconomics after the Crisis: Time to Deal with the Pretense-of-Knowledge Syndrome'. *The Journal of Economic Perspectives*, 24: 85–102.

Caiani, A., A. Godin, and S. Lucarelli (2014). 'A Stock Flow Consistent Analysis of a Schumpeterian Innovation Economy'. *Metroeconomica*, 65: 397–429.

Calvo, G.A, A. Izquierdo, and L.-F. Mejia (2004). 'On the Empirics of Sudden Stops: The Relevance of Balance-Sheet Effects'. NBER Working Paper 10520. Cambridge, MA: National Bureau of Economic Research.

Carrillo, J.A., and C. Poilly (2010). 'On the Recovery Path during a Liquidity Trap: Do Financial Frictions Matter for Fiscal Multipliers?' IRES Discussion Paper 2010034. Louvain-la-Neuve: Institut de Recherches Economiques et Sociales, Universitè catholique de Louvain.

Caverzasi, E., and A. Godin (2015). 'Post-Keynesian Stock-Flow-Consistent Modelling: A Survey'. *Cambridge Journal of Economics*, 39: 157–87.

Chen, N.-F., R. Roll, and S.A. Ross (1986). 'Economic Forces and the Stock Market'. *Journal of Business*, 59: 383–403.

Chow, G.C. (2011). 'Usefulness of Adaptive and Rational Expectations in Economics'. CEPS Working Paper 221. Princeton, NJ: Princeton University.

Christiano, L.J., M. Eichenbaum, and C.L. Evans (2005). 'Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy'. *Journal of Political Economy*, 113: 1–45.

Condon, T., H. Dahl, and S. Devarajan (1987). 'Implementing a Computable General Equilibrium Model on GAMS: The Cameroon Model'. Developmental Research Department Working Paper 290. Washington, DC: World Bank.

de Jager, S., M. Johnston, and R. Steinbach (2015). 'A Revised Quarterly Projection Model for South Africa'. South African Reserve Bank Working Paper WP/15/03. Pretoria: South African Reserve Bank.

Devarajan, S., and D.S. Go (1998). 'The Simplest Dynamic General-Equilibrium Model of an Open Economy'. *Journal of Policy Modeling*, 20: 677–714.

Dias, F., C. Duarte, and A. Rua (2010). 'Inflation Expectations in the Euro Area: Are Consumers Rational?'. *Review of World Economics/Weltwirtschaftliches Archiv*, 146: 591–607.

Duca, J., and J. Muellbauer (2014). 'Tobin LIVES: Integrating Evolving Credit Market Architecture into Flow-of-Funds Based Macro-Models'. In B. Winkler, A. van Riet, and P. Bull (eds), *A Flow-of-Funds Perspective on the Financial Crisis. Volume II: Macroeconomic Imbalances and Risks to Financial Stability*. London: Palgrave Macmillan.

Eggertsson, G.B., and P. Krugman (2012). 'Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach'. *Quarterly Journal of Economics*, 127: 1469–513.

Ellison, M., and A. Tischbirek (2014). 'Unconventional Government Debt Purchases as a Supplement to Conventional Monetary Policy'. *Journal of Economic Dynamics and Control*, 43: 199–217.

Emini, C.A., and H. Fofack (2003). *A Financial Social Accounting Matrix for the Integrated Macroeconomic Model for Poverty Analysis: Application to Cameroon with a Fixed-Price Multiplier Analysis*. Washington, DC: World Bank Publications.

Fernández-Villaverde, J. (2010). 'Fiscal Policy in a Model With Financial Frictions'. *The American Economic Review*, 100: 35–40.

Fornaro, L. (2015). 'Financial Crises and Exchange Rate Policy'. *Journal of International Economics*, 95: 202–15.

FSB (Financial Stability Board) (2016). 'Second Phase of the G20 Data Gaps Initiative (DGI-2): First Progress Report'. Available at: www.fsb.org/2016/09/second-phase-of-the-g20-data-gaps-initiative-dgi-2-first-progress-report/ (accessed 10 October 2016).

Gerali, A., S. Neri, L. Sessa, and F.M Signoretti (2010). 'Credit and Banking in a DSGE Model of the Euro Area'. *Journal of Money, Credit and Banking*, 42: 107–41.

Gertchev, N. (2007). 'A Critique of Adaptive and Rational Expectations'. *The Quarterly Journal of Austrian Economics*, 10: 313–29.

Gertler, M., and P. Karadi (2011). 'A Model of Unconventional Monetary Policy'. *Journal of Monetary Economics*, 58: 17–34.

Godley, W. (1996). 'Money, Finance and National Income Determination: An Integrated Approach'. Working Paper 167. New York: The Jerome Levy Economics Institute.

Godley, W., and M. Lavoie (2007). *Monetary Economics: An Integrated Approach to Credit, Money, Income, Production and Wealth*. Houndmills and New York: Palgrave Macmillan.

Godley, W., and M. Lavoie (2012). *Monetary Economics: An Integrated Approach to Credit, Money, Income, Production and Wealth*. New York: Springer.

Gupta, R., and M.P. Modise (2013). 'Macroeconomic Variables and South African Stock Return Predictability'. *Economic Modelling*, 30: 612–22.

Hommes, C. (2011). 'The Heterogeneous Expectations Hypothesis: Some Evidence from the Lab'. *Journal of Economic Dynamics and Control*, 35: 1–24.

Hubic, A. (2012). 'A Financial Social Accounting Matrix (SAM) for Luxembourg'. BCL Working Paper 72. Luxembourg: Central Bank of Luxembourg.

Johannsen, B.K. (2014). 'Inflation Experience and Inflation Expectations: Dispersion and Disagreement Within Demographic Groups'. Finance and Economics Discussion Series 2014-89. Washington, DC: Divisions of Research & Statistics and Monetary Affairs, Federal Reserve Board.

Kabundi, A., and E. Schaling (2013). 'Inflation and Inflation Expectations in South Africa: An Attempt at Explanation'. *South African Journal of Economics*, 81: 346–55.

Kiyotaki, N., and J. Moore (1997). 'Credit Cycles'. *Journal of Political Economy*, 105: 211–48.

Kollmann, R., M. Ratto, W. Roeger, and J. in't Veld (2013). 'Fiscal Policy, Banks and the Financial Crisis'. *Journal of Economic Dynamics and Control*, 37: 387–403.

Kreuser, F., R. Burger, and N. Rankin (2015). 'The Elasticity of Substitution and Labour Displacing Technical Change in Post-Apartheid South Africa'. Working Paper 2015/101. Helsinki: UNU-WIDER.

Mankiw, N.G., R. Reis, and J. Wolfers (2003). 'Disagreement about Inflation Expectations'. NBER Working Papers 9796. Cambridge, MA: National Bureau of Economic Research.

Monyela, C., and S. Madonsela (2015). 'Note on the Flow of Funds in South Africa's National Financial Account for the Year 2014'. *Quarterly Bulletin*, June. Pretoria: South Africa National Reserve Bank.

Ottonello, P. (2013). 'Optimal Exchange Rate Policy under Collateral Constraints and Wage Rigidity'. Mimeo. New York: Columbia University.

Patinkin, D. (1948). 'Price Flexibility and Full Employment'. *The American Economic Review*, 38: 543–64.

Rapach, D.E., M.E. Wohar, and J. Rangvid (2005). 'Macro Variables and International Stock Return Predictability'. *International Journal of Forecasting*, 21: 137–66.

Roos, M.W.M., and W.J. Luhan (2013). 'Information, Learning and Expectations in an Experimental Model Economy'. *Economica*, 80: 513–31.

Saikkonen, L. (2015). 'Estimation of Substitution and Transformation Elasticities for South African Trade'. Working Paper 2015/104. Helsinki: UNU-WIDER.

Simon, H.A. (1955). 'A Behavioral Model of Rational Choice'. *The Quarterly Journal of Economics*, 69: 99–118.

Simon, H.A. (1982). *Models of Bounded Rationality: Empirically Grounded Economic Reason*. Cambridge, MA: MIT Press.

Simon, H.A. (1986). 'Rationality in Psychology and Economics'. *The Journal of Business*, 59: S209–S24.

Sims, C. (2006). 'Comment on Del Negro, Schorfheide, Smets and Wouters'. Unpublished comment. Princeton, NJ: Princeton University. Available at:

http://sims.princeton.edu/yftp/DSSW806/DSseattleComment.pdf (accessed 29 October 2017).

Smets, F., and R. Wouters. (2003). 'An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area'. *Journal of the European Economic Association*, 1: 1123–75.

Tobin, J. (1975). 'Keynesian Models of Recession and Depression'. *The American Economic Review*, 65: 195–202.

Tobin, J. (1982). 'Money and Finance in the Macroeconomic Process'. *Journal of Money, Credit & Banking*, 14: 171–204.

Tovar, C.E. (2009). 'DSGE Models and Central Banks'. *Economics: The Open-Access, Open-Assessment E-Journal*, 3: 1–31.

Woodford, M. (2010). 'Financial Intermediation and Macroeconomic Analysis'. *Journal of Economic Perspectives*, 24: 21–44.

Appendix A: Detailed model representation

Indices			
f	Factors: labour and capital	incb	Agents ($infin \subset i$): all sectors except the Central Bank
fi	Financial instruments: cash and deposits (cd), bonds (b), equities (e), loans (l), other financial instruments (oa)	infin	Agents ($infin \subset i$): all sectors except the non-financial sector
$finb$	Financial instruments ($finb \subset fi$): all except bonds	ing	Agents ($infin \subset i$): all sectors except government
$fint$	Financial instruments ($finb \subset fi$): interest bearing	ins	Agents ($ins \subset i$): households, financial and non-financial sectors
i	Agents: households (h), financial sector (fin), non-financial sector (nfin), government (gov), Reserve Bank (rb), the rest of the world (row)	insd	Agents ($ins \subset i$): domestic sectors
ifin	Agents ($ifin \subset i$): all sectors except the financial sector	it	Agents ($it \subset i$): financial and foreign sectors
ii	Agents ($ii \subset i$): financial and non-financial sectors	le	Financial instruments ($le \subset fi$): loans and equities
il	Agents ($il \subset i$): households, non-financial sector, government		
in	Agents ($in \subset i$): non-financial and foreign sectors, government	t	Time periods
Exogenous parameters			
α_1^{gap}	Responsiveness of the demand for factors of production to real loan rates	μ_3^{peq}	Responsiveness of equity prices to economic activity
α_2^{gap}	Responsiveness of the demand for factors of production to aggregate demand	μ^{cd}	Responsiveness of the demand for cash and deposits to changes in the real return
$\alpha^{i,cd}$	Demand for cash and deposits as share of income	$\lambda_{fi,fi}^{it}$	Coefficients describing the responsiveness of the demand for assets for the foreign and financial sectors to changes in asset returns
$\alpha^{i,l}$	Demand for loans as share of income or GDP for the foreign sector	σ^{cd}	Growth of cash and deposit liabilities
α^{p}	Production function shift parameter	$dvdcoefpar$	Translation key for dividend income
α^{m}	Import function shift parameter	grg	Government consumption growth rate
α^{t}	Export function shift parameter	inf	Inflation target (steady-state inflation)

β	Discount factor for household consumption	$inta$	Quantity of aggregate intermediate input per output
β^{repo}	Responsiveness of the financial sector reserve ratio to changes in the repo rate	inv	Change in inventories
β_1^{repo}	Responsiveness of the supply of loans to changes in the repo rate	$ivat$	Quantity of aggregate output per value added
β_2^{repo}	Taylor Rule coefficient on inflation	ld	Share of wealth provided as loans for the non-financial, government, and foreign sectors
β_3^{repo}	Taylor Rule coefficient on the output gap	$lmrow$	share of foreign loan and equity liability as percentage of GDP
β^{sa}	Responsiveness of the financial sector reserve ratio to changes in the growth rate of financial assets	$mpsbar$	Steady-state savings rate
γ_1^I	Steady-state growth rate of investment	$mps01$	0–1 parameter with 1 for institutions, which marginal propensity to save adjusts
γ_2^I	Responsiveness of investment by the non-financial sector to the Tobin's Q term	n_t^i	Net change in capital transfers
γ_3^I	Responsiveness of investment to the real loan rate	$oipprm$	Other income paid as share of GDP
δ	Capital depreciation rate	pwe	World export price
δ^p	Production function share parameter	pwm	World import price
δ^q	Import function share parameter	r	Statistical residual
δ^t	Export function share parameter	res	Foreign currency reserves
θ^p	Responsiveness of price expectation to deviations of expected prices from actual prices in the previous period	sa	Stock of other assets excluding reserves
θ_1^{pq}	Responsiveness of prices to changes in the output gap	sl	Stock of equity liabilities of the financial sector other than interests in retirement and life funds
θ_2^{pq}	Responsiveness of prices to changes in import prices	slr	Other liabilities for the foreign sector (excluding foreign currency reserves)
θ^{prce}	Responsiveness of price expectations to the output gap	$shifint$	Share of interest income
ρ^{lab}	Labour force growth	$shifoint$	Share of other income
ρ^p	Production function substitution elasticity	$shifsoc$	Share of social contributions received
ρ^q	Import function substitution elasticity	shr	Share of interest income
ρ^{repo}	Interest rate smoothing coefficient	soc_par	Social contributions paid as a share of GDP

ρ^t	Export function substitution elasticity	ss^{peq}	Steady-state growth in equity prices
μ^{cd}	Responsiveness of the demand for cash and deposits to changes in the real return	ta	Activity tax rate
μ^{fw}	Target real growth rate for household wealth	td	Personal direct tax rate
μ^l	Responsiveness of the demand for loans to changes in real borrowing costs	tm	Import tariff rate
μ_1^{peq}	Responsiveness of equity prices to inflation	ts	Sales tax rate
μ_2^{peq}	Responsiveness of equity prices to money supply	$yfrow$	Factor income payments to the foreign sector
μ_2^{rcd}	Coefficient in the cash and deposit rate equation	$yfmrow$	Factor income received from the foreign sector
Endogenous variables			
$BSASS$	Stock of assets	NOI	Net other income inflows
$BSLIB$	Stock of liabilities	$NINT$	Net interest income inflows
d_as_b	Change in the stock of bonds (assets)	NW	Net wealth of institutions
d_as_cd	Change in the stock of cash and deposits (assets)	OI	Other income
d_as_e	Change in the stock of equities (assets)	OIP	Other payments
d_as_l	Change in the stock of loans (assets)	PA	Aggregate sector output price
d_as_oa	Change in the stock of other (assets)	PD	Domestic supply price with margin
d_lib_b	Change in the stock of bonds(liabilities)	PE	Export price
d_lib_cd	Change in the stock of cash and deposits (liabilities)	PEQ	Equity price
d_lib_e	Change in the stock of equities (liabilities)	PM	Import price
d_lib_l	Change in the stock of loans (liabilities)	PQ	Composite supply price
d_lib_oa	Change in the stock of other (liabilities)	PRC	Price including sales taxes
$DFRATEB$	Bond rate-adjustment factor	$PRCd$	Inflation
$DFRATECD$	Cash and deposit rate-adjustment factor	$PRCe$	Expected prices
$DFRATEL$	Loan rate-adjustment factor	PVA	Composite value-added price
DVD	Dividend income	QA	Aggregate output quantity
$DVDCOEF$	Share of dividend income	QD	Domestic supply quantity
$DVDFIN$	Dividends paid by the financial sector	QE	Export quantity
$DVDP$	Dividends paid	QF	Factor demand
$DVDROW$	Dividends paid by the foreign sector	QFS	Factor supply
EXR	Exchange rate	QG	Government consumption quantity

$FINT$	Interest income paid to the rest of the world	QH	Household consumption quantity
$FRATEB$	Bond rate	$QINTA$	Intermediate input
$FRATECD$	Cash and deposit rate	QM	Import quantity
$FRATEL$	Loan rate	QQ	Composite supply quantity
$FREPO$	Repo rate	QVA	Composite value-added quantity
FW	Financial wealth	RR	Reserve ratio
$FW1$	Real financial wealth	SAV	Savings of agents
$GGAP$	Factor demand adjustment factor (proxy for output gap)	$SAVF$	Foreign savings
$GDPMP$	Real GDP	$SOCCONTP$	Social contributions paid
$INTINCT$	Interest income	$SOCCONTR$	Income from social contribution
$INTEXPBOND$	Real interest expenditure bonds	U	Household utility
$INTEXP1$	Real interest expenditure	WF	Economy-wide factor return
INV	Investment	YF	Total factor income
MPS	Marginal propensity to save	YI	Real agent income
$NDVD$	Net dividend inflows	YIF	Factor income

Prices and interest rates

$$PM_t = pwm_t \cdot (1 + tm_t) \cdot EXR_t \quad \text{A1}$$

$$PE_t = pwe_t \cdot (1 - te_t) \cdot EXR_t \quad \text{A2}$$

$$PQ_t = (1 + inf) \cdot PQ_{t-1} + \theta_1^{pq} \cdot \left(GGAP_{t-1}^{l} - 1\right) + \theta_2^{pq} \cdot \Delta PM_t \quad \text{A3}$$

$$PQ_t \cdot QQ_t = PD_t \cdot QD_t + PM_t \cdot QM_t \quad \text{A4}$$

$$PA_t \cdot QA_t = PDD_t \cdot QD_t + PE_t \cdot QE_t \quad \text{A5}$$

$$PRC_t = (1 + ts_t) \cdot PQ_t \quad \text{A6}$$

$$PRCd_t = \Delta PRC_t \quad \text{A7}$$

$$PRC_t{}^{e} = (1 + inf) \cdot PRC_{t-1}{}^{e} - \theta^{p} \cdot (PRC_{t-1}{}^{e} - PRC_{t-1}) + \theta^{prce} \cdot \left(GGAP_{t-1}^{l} - 1\right) \quad \text{A8}$$

$$PEQ_t = \left((1 + ss^{peq}) \cdot (1 + PRCd_t^{e})^{\mu_1^{peq}} \cdot (\Delta SL_t^{fin\_cd})^{\mu_2^{peq}} \cdot (\Delta QA_t)^{\mu_3^{peq}}\right) \cdot PEQ_{t-1} \quad \text{A9}$$

$$FREPO_t = 0.7 \cdot FREPO_{t-1} + 0.3 \cdot (inf + 2.0 \cdot (PRCd_t - inf) + 0.35 \cdot (GGAP_{t-1}^{l})) \quad \text{A10}$$

$$FRATEB_t = FREPO_t + DFRATEB_t \quad \text{A11}$$

$$FRATECD_t = 0.6 \cdot FREPO_t + 0.4 \cdot FRATEL_t \quad \text{A12}$$

$$FRATEL_t = FREPO_t + DFRATEL_t \quad \text{A13}$$

Production and trade

$$QVA_{cst} = \alpha_{cst}^p \cdot \sum_f \left(\delta_{fcs}^p \cdot QF_{fcst}^{-\rho_{cs}^p}\right)^{-1/\rho_{cs}^p} \quad \text{A14}$$

$$WF_{ft} = PVA_t \cdot QVA_t \cdot \sum_{f'} \left(\delta_f^p \cdot QF_{ft}^{-\rho^p}\right)^{-1} \cdot \delta_f^p \cdot QF_{ft}^{-\rho^p-1} \quad \text{A15}$$

$$QINTA_t = inta \cdot QA_t \quad \text{A16}$$

$$QVA_t + QINTA_t = QA_t \quad \text{A17}$$

$$QA_t = \alpha^t \cdot \left(\delta^t \cdot QE_t^{\rho^t} + (1-\delta^t) \cdot QD_t^{\rho^t}\right)^{1/\rho^t} \quad \text{A18}$$

$$\frac{QE_t}{QD_t} = \left(\frac{PE_t}{PDD_t} \cdot \frac{(1-\delta^t)}{\delta^t}\right)^{1/(\rho^t-1)} \quad \text{A19}$$

$$QQ_t = \alpha^q \cdot \left(\delta^q \cdot QM_t^{-\rho^q} + (1-\delta^q) \cdot QD_t^{-\rho^q}\right)^{-1/\rho^t} \quad \text{A20}$$

$$\frac{QM_t}{QD_t} = \left(\frac{PDD_t}{PM_t} \cdot \frac{(1-\delta^q)}{\delta^q}\right)^{1/(1+\rho^t)} \quad \text{A21}$$

Incomes and expenditures

$$YF_{ft} = WF_{ft} \cdot QF_{ft} + EXR_t \cdot yfmrow_t \quad \text{A22}$$

$$YIF_{ft}{}^i = shif^i \cdot (YF_{ft} - EXR_t \cdot yfrow_t)$$

$$DVDP_t^{ii} = (1 - td_t^{ii}) \cdot PRC_t \cdot YI_t^{ii} - SC_t^{ii} - OIP_t^{ii} - IEXP_t^{ii} - SAV_t^{ii} \quad \text{A23}$$

$$PRC_t \cdot INTEXP1_t^i = FRATEL_{t-1} \cdot BSLIB_{t-1}^{i,l} + FRATECD_{t-1} \cdot BSLIB_{t-1}^{i,cd} + INTEXPBOND_t^i \quad \text{A24}$$

$$INTEXPBOND_t^i = FRATEB_{t-1} \cdot d\_lib_{t-1}^{i,b} + INTEXPBOND_{t-1}^i \quad \text{A25}$$

$$DVD_t^i = DVDCOEF_t^i \cdot \left(\sum_{ii} DVDP_t^{ii} + EXR_t \cdot DVDROW_t\right) \quad \text{A26}$$

$$INT_t^i = INTCOEF_t^i \cdot \sum_i PRC_t \cdot INTEXP1_t^i \quad \text{A27}$$

$$OI_t^i = shifoint^i \cdot \sum_i OIP_t^i \quad \text{A28}$$

$$SOCCONTR_t^i = shifosoc^i \cdot \sum_i SOCCONTP_t^i \quad \text{A29}$$

$$OIP_t^i = oipprm^i \cdot GDPMP_{t-1} \quad \text{A30}$$

$$SOCCONTP_t^i = soc\_par^i \cdot GDPMP_{t-1} \quad \text{A31}$$

$$DVDCOEF_t^{insd} = dvdcoefpar^{insd} \cdot \frac{BSASS_{t-1}^{insd,e}}{\sum_{insd} BSASS_{t-1}^{insd,e}} \quad \text{A32}$$

$$DVDCOEF_t^{row} = 1 - \sum_{isnd} DVDCOEF_t^{insd} \quad \text{A33}$$

$$INTCOEF_t^{insd} = intcoefpar^{insd} \cdot \frac{\sum_{fint} BSASS_{t-1}^{insd,fint}}{\sum_{fint,insd} BSASS_{t-1}^{insd,fint}} \quad \text{A34}$$

$$INTCOEF_t^{row} = 1 - \sum_{isnd} INTCOEF_t^{insd} \quad \text{A35}$$

$$PRC_t \cdot YI_t^i = \sum_f YIF_t^{i,f} + DVD_t^i + INT_t^i + OI_t^i + SOCCONTR_t^i \quad \text{A36}$$

$$U_0 = \sum_{t=0}^{10} \beta^t \, Ln(QH_t) \quad \text{A37}$$

$$INV_t^{nfin} = INV_{t-1}^{nfin} \cdot \gamma_1^I \cdot \left(\frac{\Delta PEQ_t}{\Delta PRC_t}\right)^{\gamma_2^I} \cdot \left(\frac{1 + FRATEL_t}{1 + PRCd_t^e}\right)^{\gamma_3^I} \quad \text{A38}$$

$$INV_t^{infin} = INV_{t-1}^{infin} \cdot \gamma_1^I \cdot \left(\frac{1 + FRATEL_t}{1 + PRCd_t^e}\right)^{\gamma_3^I}$$

$$QG_t = grg_t \cdot QG_{t-1} \quad \text{A39}$$

$$GDPMP_t = QH_t + QG_t + \sum_i INV_i^i + inv + QE_t - QM_t + r \quad \text{A40}$$

$$PRC_t \cdot YI_t^{gov} = \sum_f YIF_t^{gov,f} + DVD_t^{gov} + INT_t^{gov} + OI_t^{gov} + SOCCONTR_t^{gov} + ts_t \cdot PQ_t \cdot QQ_t + ta_t QQ_t + ta_t \cdot PA_t \cdot QA_t + tm_t \cdot pwm_t \cdot EXR_t \cdot QM_t + \sum_{insd} td_t \cdot PRC_t \cdot YI_t^{insd} \quad \text{A41}$$

Equilibrium conditions

$$QF_t^f = GGAP_t \cdot QFS_t^f \quad \text{A42}$$

$$QQ_t = QINTA_t + QH_t + QG_t + \sum_i INV_t^i + inv \quad \text{A43}$$

$$GGAP_t^f = GGAP_{t-1}^f - \alpha_1^{gap,f} \cdot (FRATEL_{t-1} - PRCd_{t-1}) + \alpha_2^{gap,f}(\Delta AD_{t-1} - \Delta ad) \quad \text{A44}$$

$$SAVF_t = pwm_t \cdot QM_t - pwm_t \cdot QE_t - TRANSF_t \quad \text{A45}$$

$$TRANSF_t = nyif + NDVD_t + NOI_t + NINT_t \quad \text{A46}$$

$$\sum_i BSASS_t^{i,fi} = \sum_i BSLIB_t^{i,fi} \quad \text{A47}$$

$$d\_lib_t^{gov,b} = PRC_t \cdot INV_t^{gov} + \sum_{fib} d\_as_t^{gov,fib} + PEQ_t \cdot d\_as_t^{gov,e} - n_t^{gov} - SAV_t^{gov} - \sum_{fi} d_{lib}{}_t^{gov,fi} - PEQ_t \sum_{fi} d_{lib}{}_t^{gov,fi} - PEQ_t \cdot d\_lib_t^{gov,e} \quad \text{A48}$$

$$\sum_i SAV_i^i + EXR_t \cdot SAVF_t = \sum_i INV_i^i + inv \quad \text{A49}$$

$$\frac{SAV_t^H}{(1 - td_t^H) \cdot PRC_t \cdot YI_t^H} = \frac{SAV_{t-1}^H}{(1 - td_{t-1}^H) \cdot PRC_{t-1} \cdot YI_{t-1}^H} \quad \text{A50}$$

Factor accumulation

$$QFS_t^k \; QFS_t^k = (1 - \delta) \cdot QFS_{t-1}^k + \sum_i INV_{it-1} + inv \quad \text{A51}$$

$$QFS_t^l \; QFS_t^l = (1 + \rho^{lab}) \cdot QFS_{t-1}^l \quad \text{A52}$$

Savings

$$SAV_t^{CB} = (1 - td_t^{CB}) \cdot PRC_t \cdot YI_t^{CB} - IEXP_t^{CB}$$ A53

$$SAV_t^{H} = (1 - td_t^{H}) \cdot PRC_t \cdot YI_t^{H} - SOCCONTP_t^{H} - OIP_t^{H} - PRC_t \cdot INTEXP1_t^{H} - PRC_t \cdot QH_t$$ A54

$$SAV_t^{gov} = PRC_t \cdot YI_t^{gov} - PRC_t \cdot QG_t - SOCCONTP_t^{gov} - OIP_t^{gov} - PRC_t \cdot INTEXP1_t^{gov}$$ A55

$$MPS_t^{ii} = mpsbar^{ii}(1 + MPSADJ_t \cdot mps01_t^{ii})$$ A56

$$SAV_t^{ii} = MPS_t^{ii}(1 - td_t^{i}) \cdot PRC_t \cdot YI_t^{ii}$$ A57

Financial side

$$FW_t^i = \sum_{fi} BSASS_{t-1}^{i,fi} + PEQ_t \cdot BSASS_{t-1}^{i,eq} + \sum_{fi} d\_lib_t^{i,fi} + PEQ_t \cdot d\_lib_t^{i,eq} + SAV_t^i - PRC \cdot INV_t^i + n_t^i$$ A58

$$FW1_{t,t+10}^{H} = FW1_{t-1}^{H} \cdot \left(1 + \mu^{fw}\right)^{10}$$ A59

$$FW1_t^i = \frac{FW_t^i}{PRC_t}$$ A60

$$FRATEEQ_t = \frac{DVD_{t-1}^{fin}}{PEQ_{t-1} \cdot BSASS_{t-1}^{fin,eq}}$$ A61

$$BSASS_t^{it,fi} = (FW_t^{iii} - BSLIB_t^{iii,l}) \cdot \left(\lambda_{f0} + A_{fi,j} \cdot r_{fi,j} + \lambda_{fi,4} \frac{\cdot PRC_t \cdot YI_t^{it}}{FW_t^{it}}\right)$$ A62

$$PEQ_t \cdot BSASS_t^{h,e} = FW_t^{h} - BSASS_t^{h,e} - BSASS_t^{h,oa} - BSASS_t^{h,l} - BSASS_t^{h,b}$$ A63

$$BSASS_t^{fin,loans} = (1 - RR_t) \cdot BSLIB_{t-1}^{fin,cashdep}$$ A64

$$BSASS_t^{in,l} = ld^{in} \cdot (1 + FREPO_t)^{\beta_1^{repo}} \cdot FW_t^{in}$$ A65

$$RR_t = RR_{t-1} \cdot (1 + \Delta FREPO_t)^{\beta^{repo}} \cdot \left(\frac{\sum_{fi} BSASS_t^{fin,fi} + PEQ_t \cdot BSASS_t^{fin,eq}}{\sum_{fi} BSASS_{t-1}^{fin,fi} + PEQ_t \cdot BSASS_{t-1}^{fin,eq}}\right)^{\beta^{sa}}$$ A66

$$BSASS_t^{il,cd} = \alpha^{il,cd} \cdot \left(\frac{1 + FRATECD_t}{1 + PRCd_t}\right)^{\mu^{cd}} \cdot PRC_t \cdot YI_t^{il}$$ A67

$$BSASS_t^{rb,oa} = EXR_t \cdot res + sa^{rb,oa}$$ A68

$$BSASS_t^{rb,b} = BSASS_{t-1}^{rb,b} + d\_as_t^{rb,b}$$ A69

$$BSLIB_t^{row,oa} = EXR_t \cdot res + slr^{oa}$$ A70

$$BSLIB_t^{ins,l} = \alpha^{ins,l} \cdot \left(\frac{1 + FRATEL_t}{1 + PRCd_t}\right)^{\mu^{l}} \cdot PRC_t \cdot YI_t^{ins}$$ A71

$$BSLIB_t^{row,le} = lmrow^{le} \frac{\cdot PRC_{t-1} \cdot GDPMP_{t-1}}{EXR_{t-1}}$$ A72

$$BSLIB_t^{fin,eq} = sl^{fin,e} + BSASS_t^{H,eq}$$ A73

$$BSLIB_t^{nfin,eq} = \sum_i BSASS_t^{i,eq} - \sum_{infin} BSLIB_t^{infin,eq}$$ A74

$$BSLIB_t^{cb,cd} = \sigma^{cd} \cdot \sum_i PRC_t \cdot YI_t^i$$ A75

$$BSLIB_t^{fin,cd} = \sum_i BSASS_t^{i,cd} - \sum_{ifin} BSLIB_t^{ifin,cd}$$ A76

$$BSLIB_t^{gov,b} = BSLIB_{t-1}^{gov,b} + d\_lib_t^{gov,b}$$ A77

$$d\_lib_t^{ing,b} = BSLIB_t^{ing,b} - BSLIB_{t-1}^{ing,b}$$ A78

$$d\_as_t^{incb,b} = BSASS_t^{incb,b} - BSASS_{t-1}^{incb,b}$$ A79

$$\begin{aligned} d\_as_t^{rb,b} = {} & SAV_t^{rb} + \sum_{fi} d\_lib_t^{rb,fi} + PEQ_t \cdot d\_lib_t^{rb,e} - PEQ_t \cdot d\_as_t^{rb,e} \\ & - \sum_{fib} d\_as_t^{rb,fib} - PRC_t \cdot INV_t^{rb} + n_t^{cb} \end{aligned}$$ A80

$$d\_lib_t^{i,finb} = BSLIB_t^{i,finb} - BSLIB_{t-1}^{i,finb}$$ A81

$$d\_as_t^{i,finb} = BSASS_t^{i,finb} - BSASS_{t-1}^{i,finb}$$ A82

$$\begin{aligned} NW_t^i &= \frac{\left(\sum_{fi} BSASS_t^{i,fi} + PEQ_t \cdot BSASS_t^{i,e} - \sum_{fi} BSLIB_t^{i,fi} - PEQ_t \cdot BSLIB_t^{i,e}\right)}{PRC_t \cdot GDPMP_t} NW_t^i \\ &= \frac{\left(\sum_{fi} BSASS_t^{i,fi} + PEQ_t \cdot BSASS_t^{i,e} - \sum_{fi} BSLIB_t^{i,fi} - PEQ_t \cdot BSLIB_t^{i,e}\right)}{PRC_t \cdot GDPMP_t} \end{aligned}$$ A83

Source: Authors' own elaboration.

Appendix B: Exogenous coefficients

Table: Exogenous parameters

$\alpha_1^{gap,l}$ $\alpha_1^{gap,k}$	Responsiveness of the demand for factors of production to real loan rates	0.45 0.40	μ^l	Responsiveness of the demand for loans to changes in the real borrowing costs	−5
$\alpha_2^{gap,l}$ $\alpha_2^{gap,k}$	Responsiveness of the demand for factors of production to aggregate demand	0.35 0.3	μ_1^{peq}	Responsiveness of equity prices to inflation expectations	0.55
$\alpha^{mgov,cd}$	Demand for cash and deposits as share of income: government sector	0.13	μ_2^{peq}	Responsiveness of equity prices to money supply	0.2
$\alpha^{mhhd,cd}$	Demand for cash and deposits as share of income: household	0.34	μ_3^{peq}	Responsiveness of equity prices to economic activity	0.33
$\alpha^{nfin,cd}$	Demand for cash and deposits as share of income: non-financial sector	1.16	μ_2^{rcd}	Coefficient in the cash and deposit rate equation	0.6
$\alpha^{cb,l}$	Demand for loans as share of income: Reserve Bank	12.7	$\lambda_{b,0}^{fin}$ $\lambda_{b,0}^{mrow}$	Tobin demand coefficient showing the steady-state share of bonds in total wealth (fin: financial sector, $mrow$: foreign sector)	0.22 0.18
$\alpha^{fin,l}$	Demand for loans as share of income: financial sector	0.31	$\lambda_{eq,0}^{fin}$ $\lambda_{eq,0}^{mrow}$	Tobin demand coefficient showing the steady-state share of equities in total wealth	0.52 0.73
$\alpha^{mgov,l}$	Demand for loans as share of income: government sector	0.25	$\lambda_{cd,0}^{fin}$ $\lambda_{cd,0}^{mrow}$	Tobin demand coefficient showing the steady-state share of cash and deposits in total wealth	0.26 0.1
$\alpha^{mhhd,l}$	Demand for loans as share of income: household sector	0.61	$\lambda_{b,1}^{fin}$ $\lambda_{b,1}^{mrow}$	Tobin demand coefficient: responsiveness of bond to changes in the bond return	1.99 1.99
$\alpha^{mrow,l}$	Demand for loans as share of GDP: foreign sector	0.12	$\lambda_{b,2}^{fin}$ $\lambda_{b,2}^{mrow}$	Tobin demand coefficient: responsiveness of bond to changes in the equity return	−1.6 −1.6
$\alpha^{nfin,l}$	Demand for loans as share of income: non-financial sector	1.26	$\lambda_{b,3}^{fin}$ $\lambda_{b,3}^{mrow}$	Tobin demand coefficient: responsiveness of bond to changes in the cash and deposit return	−0.2 −0.2
α^p	Production function shift parameter (base year)	0.84	$\lambda_{b,4}^{fin}$ $\lambda_{b,4}^{mrow}$	Tobin demand coefficient: responsiveness of bond to changes in transactional demand for money	0 0
α^m	Import function shift parameter	1.32	$\lambda_{eq,1}^{fin}$ $\lambda_{eq,1}^{mrow}$	Tobin demand coefficient: responsiveness of equity to changes in the bond return	−1.3 −1.3
α^t	Export function shift parameter	5.55	$\lambda_{eq,2}^{fin}$ $\lambda_{eq,2}^{mrow}$	Tobin demand coefficient: responsiveness of equity to changes in the equity return	1.6 1.6
β^{repo}	Responsiveness of the financial sector reserve ratio to changes in the repo rate	15.3	$\lambda_{eq,3}^{fin}$ $\lambda_{eq,3}^{mrow}$	Tobin demand coefficient: responsiveness of equity to changes in the cash and deposit return	−0.6 −0.6

β_1^{repo}	Responsiveness of the supply of loans to changes in the repo rate	−0.7	$\lambda_{eq,4}^{fin}$ $\lambda_{eq,4}^{mrow}$	Tobin demand coefficient: responsiveness of equity to changes in transactional demand for money	−0.001 −0.001
β_2^{repo}	Taylor Rule coefficient on inflation	2.0	$\lambda_{cd,1}^{fin}$ $\lambda_{cd,1}^{mrow}$	Tobin demand coefficient: responsiveness of cash and deposit to changes in the bond return	−0.66 −0.66
β_3^{repo}	Taylor Rule coefficient on the output gap	0.3	$\lambda_{cd,2}^{fin}$ $\lambda_{cd,2}^{mrow}$	Tobin demand coefficient: responsiveness of cash and deposit to changes in the equity return	−0.04 −0.04
β^{sa}	Responsiveness of the financial sector reserve ratio to changes in the growth rate of financial assets	−0.1	$\lambda_{cd,3}^{fin}$ $\lambda_{cd,3}^{mrow}$	Tobin demand coefficient: responsiveness of cash and deposit to changes in the cash and deposit return	0.7 0.7
γ_1^I	Steady-state growth rate of investment	1.04	$\lambda_{cd,4}^{fin}$ $\lambda_{cd,4}^{mrow}$	Tobin demand coefficient: responsiveness of cash and deposit to changes in transactional demand for money	0.001 0.001
γ_2^I	Responsiveness of investment by the non-financial sector to the Tobin's Q term	0.14	σ^{cd}	Growth of cash and deposit liabilities of the Reserve Bank coefficient	0.027
γ_3^I	Responsiveness of investment to the real loan rate	−0.9	grg	Government consumption growth rate	0.026
δ	Capital depreciation rate	0.5	inf	Inflation target (steady-state inflation)	0.06
δ^p	Production function share parameter	0.0001	$inta$	Quantity of aggregate intermediate input per output	0.06
δ^q	Import function share parameter	0.03	$ivat$	Quantity of aggregate output per value added	0.44
δ^t	Export function share parameter	0.99	ld^{mgov} ld^{mrow} ld^{nfin}	Share of wealth provided as loans for the non-financial, government, and foreign sectors	0.421 0.065 0.385
θ^p	Responsiveness of price expectation to deviations of expected prices from actual prices in the previous period	0.2	$lmrow$	Share of foreign loan and equity liability as percentage of GDP	0.18
θ_1^{pq}	Responsiveness of prices to changes in the output gap	1.5	$mpsbar^{fin}$ $mpsbar^{nfin}$	Steady-state savings rate	0.07 0.44
θ_2^{pq}	Responsiveness of prices to changes in import prices	0.5	$oipprm^{fin}$ $oipprm^{mhhd}$ $oipprm^{mgov}$ $oipprm^{mrow}$ $oipprm^{nfin}$	Other income paid as share of GDP	0.15 0.04 0.01 0.001 0.01
θ^{prce}	Responsiveness of price expectations to the output gap	0.3	$shifoint^{fin}$ $shifoint^{mhhd}$ $shifoint^{mgov}$ $shifoint^{mrow}$ $shifoint^{nfin}$	Share of other income received	0.23 0.68 0.01 0.03 0.06
ρ^{lab}	Labour force growth	0.01	$shifsoc^{fin}$ $shifsoc^{mhhd}$ $shifsoc^{mgov}$	Share of social contributions received	0.51 0.46 0.03
ρ^p	Production function substitution elasticity	−1	soc_par^{fin} soc_par^{mhhd}	Social contributions paid as a share of GDP	0.05 0.09

			soc_par^{mgov} soc_par^{nfin}		0.03 0.004
ρ^{q}	Import function substitution elasticity	1	ss^{peq}	Steady-state growth in equity prices	0.03
ρ^{repo}	Interest rate smoothing coefficient	0.7	ta	Activity tax rate	0.009
ρ^{t}	Export function substitution elasticity	11	td	Personal direct tax rate	0.009
μ^{cd}	Responsiveness of the demand for cash and deposits to changes in the real return	0.2	tm	Import tariff rate	0.027
μ^{fw}	Target real growth rate for household wealth	0.15	ts	Sales tax rate	0.038

Source: Authors' own elaboration.